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The Role of Common Cyclical Features for Coincident and Leading Indexes Building

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Abstract

In this paper we propose a new methodology to build composite coincident and leading indexes. Based on a formal definition which requires that the first difference of the leading index is the best linear predictor of the first difference of the coincident index, we show that the notion of polynomial serial correlation common features can be used to build these composite variables. Concepts and methods are illustrated by an empirical investigation of the US business cycle movements.

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1 Introduction

For several decades most countries have experienced a system of coincident and leading indexes. Whether the goal is to predict the turning points of the economic activity or the movements in the overall business cycle, the common feature of the different methodologies consists generally of two steps.

The first step aims at identifying groups of variables that move in, before of after the recession. The early NBER attempts to monitor the US business cycle with the pioneer work of Burns and Mitchell (1946) focused on the distinction between these categories (see Niemera and Klein, 1994 for a survey). We emphasize in this paper the first two groups of variables which are commonly defined as respectively the coincident and the leading indicators.

The second step is linked to the need to find parsimonious (statistical) methods for studying the relationships between the multitude of potential coincident and leading variables and consequently to facilitate the prediction of the business cycle. The expression coincident and leading indexes, noted respectively CI and LI in the rest of the paper, usually refers to the combinations of the individual indicators when forming synthetic indexes. Among the statistical approaches, the methodology developed by Stock and Watson (1989, 1991, 1993) for the NBER became in the past few years a standard reference for constructing the so called experimental indexes. But other approaches exist since a while, from the well known principal component or classical linear time series analyses to more complex non-linear modeling such as switching regimes approaches, smooth transition regressions, probit models or nonparametric methods (see Camacho and Perez-Quiros, 2002, for a comparison of the forecasting performances of these procedures).

The viewpoint in this article is that linear systems with a deeper emphasis of their dynamic properties are worth considering for constructing such indexes. Indeed, some methods for building a CI and LI pair require that the individual indicators are classified as coincident or leading according to *a priori* economic intuitions which may not be validated by the data. Afterwards, these indicators are mechanically passed through some algorithm for extracting the composite indexes. For instance, in dynamic factor type modeling, a unique factor is extracted to form the CI without a formal statistical evaluation of this structure. Consequently, the time series properties of the data are often arbitrary fixed and some important characteristics, such as the presence of cointegration, are rarely considered.

In a similar spirit as Emerson and Hendry (1996), we present a method for building CI&LI's which is based on a formal statistical analysis of the comovements among the individual economic indicators. We propose to use a variant of the notion of Serial Correlation Common Feature (SCCF, Engle and Kozicki, 1993; Vahid and Engle, 1993) to extract a coincident and a leading index from a vector of cointegrated time series. Although the SCCF is not endowed with a structural interpretation (Lippi and Reichlin, 1994) and does not reflect high coherence at the business cycle frequencies (Cubadda, 1999), ignoring its presence in VAR modeling may lead to considerable losses in forecast accuracy (Vahid and Issler, 2002). In particular, the dynamic properties of the data are investigated within the Polynomial Serial Correlation Common Feature modeling (PSCCF, Cubadda and Hecq, 2001). Indeed, this approach is more appropriate for the analysis of non-contemporaneous short-run comovements than the original SCCF modeling.

Previewing the main results, the relevant characteristics of our composite indicators are that 1°) the changes of the LI are the best linear predictor of the changes of the CI, 2°) the existence of such CI&LI is tested and not assumed *a priori*, 3°) there is no need of clustering the individual indicators in leading or coincident, 4°) statistical inference is easily performed by reduced rank regression or FIML, and 5°) parameter stability is easily checked because only a few parameters are involved under the PSCCF compared to the VAR.

This paper is organized as follows. Section 2 summarizes the PSCCF approach, proposes a definition of the CI&LI, and shows how to build such indexes. In Section 3 the conditions for the existence of a long leading index are examined. In Section 4 the methodology is applied to the US business cycles indicators. Section 5 concludes.

2 The statistical methodology

The objective of this section is to present a framework to build the CI&LI from a set of cointegrated time series.

2.1 Preliminaries

Let us start with the VAR(p) model for a n-vector of I(1) time series $\{y_t, t = 1, \ldots, T\}$,

$$A(L)y_t = \varepsilon_t, \tag{1}$$

for fixed values of $y_{-p+1}, ..., y_0$ and where $A(L) \equiv I_n - \sum_{i=1}^{p} A_i L^i$, and ε_t are i.i.d. $N_n(0, \Sigma_\varepsilon)$ errors. To simplify the notation, we omit the deterministic terms at this stage.

We further assume that the process y_t is cointegrated of order (1,1), namely that 1°) $rank(A(1)) = r$, $0 < r < n$, so that $A(1)$ can be expressed as $A(1) = -\alpha\beta^{'}$ with α and β both $(n \times r)$ matrices of full column rank r, and 2°) the matrix $\alpha'_{\perp} A^*(1)\beta_{\perp}$ has rank equal to $(n-r)$ where $A^*(1)$ denotes the first derivative of $A(z)$ at $z = 1$. The columns of β span the space of cointegrating vectors, and the elements of α are the corresponding adjustment coefficients. In order to rewrite the system in a VECM form we use the identity $A(L) \equiv A(1)L + \Gamma(L)\Delta$ where $\Gamma(L) = I_n - \sum_{i=1}^{p-1} \Gamma_i L^i$, and $\Gamma_i = -\sum_{j=i+1}^{p} A_j$ for $i = 1, \ldots, p-1$. And finally we obtain

$$\Gamma(L)\Delta y_t = \alpha\beta' y_{t-1} + \varepsilon_t. \tag{2}$$

The stationary process Δy_t admits the following Wold representation

$$\Delta y_t = C(L)\varepsilon_t, \tag{3}$$

with $\sum_{j=1}^{\infty} j\,|C_j| < \infty$, and $C_0 = I_n$.

Based on the polynomial factorization $C(L) = C(1) + \Delta C^*(L)$, where $C_i^* = -\sum_{i+1}^{\infty} C_j$ for $i \geq 0$, we obtain the multivariate Beveridge and Nelson (1981, BN henceforth) representation of the series y_t

$$y_t = \tau_t + \xi_t, \tag{4}$$

where $\xi_t = C^*(L)\varepsilon_t$, and $\Delta\tau_t = C(1)\varepsilon_t$.

The multivariate BN decomposition has a natural interpretation in forecasting terms. Indeed, we get from Equation (4) that

$$E(y_{t+h}|\Omega_t) = \tau_t + E(\xi_{t+h}|\Omega_t), \quad h \geq 1, \tag{5}$$

where Ω_t is the σ-field generated by $\{y_{t-i}; i \geq 0\}$. Since ξ_t is a stationary process by construction, we see that $\lim_{h\to\infty} E(y_{t+h}|\Omega_t) = \tau_t$. Based on the popular view that the trend of a non-stationary time series coincides with its infinite-step ahead prediction (see e.g. Harvey, 1990), the processes τ_t and ξ_t are respectively defined as the stochastic trends and cycles of variables y_t.

In order to analyze non-contemporaneous short-run comovements, Cubadda and Hecq (2001) have introduced the notion of Polynomial Serial Correlation Common Features such that

Definition 1 ***Polynomial Serial Correlation Common Features of order m:*** *series Δy_t have s PSCCF of order m, henceforth PSCCF(m), iff there exists a $n \times s$ polynomial matrix $\delta(L) = \delta_0 - \sum_{i=1}^{m} \delta_i L^i$ with $m < (p-1)$ such that the matrix δ_0 is full column rank, $\delta_m \neq 0$, and $\delta(L)'\Delta y_t = \delta_0' \varepsilon_t$.*

Notice that the usual SCCF is obtained as a special case of the PSCCF(m) with $m = 0$.

The presence of the PSCCF(m) endows series y_t with several interesting properties. First, the following restrictions on the VECM (2) parameters hold

$$\text{Condition 1.} \qquad \delta_0'\alpha = 0$$

$$\text{Condition 2.} \qquad \delta_0'\Gamma_i = \begin{cases} \delta_i' & if\ i \leq m \\ 0 & if\ i > m \end{cases}$$

Second, variables y_t must share at least one common trend since Condition 1 implies that the matrix α has rank less then n. Third, the multivariate BN cycles ξ_t respect the following condition

$$E(\delta(L)'\xi_{t+h}|\Omega_t) = 0, \quad h \geq m, \tag{6}$$

which suggests that the PSCCF(m) may be seen as a dynamic extension of the codependent cycles modelling proposed by Vahid and Engle (1997).[1]

Statistical inference on the existence of such PSCCF(m) may be easily obtained. Indeed, since Definition 1 requires that the series $\delta(L)'\Delta y_t$ are innovations with respect to the past information, a maximum likelihood approach requires to solve the following canonical correlation program,

$$CanCor\left\{ \begin{pmatrix} \Delta y_t \\ -\Delta y_{t-1} \\ \vdots \\ -\Delta y_{t-m} \end{pmatrix}, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid D_t \right\}, \tag{7}$$

[1] Indeed, Vahid and Engle (1997) showed that the same linear combination that renders the series Δy_t a VMA(m) process transforms the cycles ξ_t into a VMA$(m-1)$ one.

where $CanCor(Y, X \mid Z)$ denotes the partial canonical correlations between the elements of Y and X conditional on Z, $\hat{\beta}$ is a superconsistent estimate of the cointegrating vectors, and D_t is a vector of deterministic terms.

The likelihood ratio (LR) test for the null hypothesis that there exist at least s PSCCF vectors is based on the statistic (see *inter alia* Anderson, 1984; Velu *et al.,* 1986)

$$LR_0 = -T \sum_{i=1}^{s} \ln(1 - \hat{\lambda}_i), \qquad s = 1, \ldots, n - r \tag{8}$$

where $\hat{\lambda}_i$ is the $i-$th smallest squared canonical correlation coming from the solution of (7). The test statistic (8) follows asymptotically a $\chi^2_{(d)}$ distribution under the null where $d = s \times (n(p - 2 - m) + r + s)$. Moreover, the eigenvectors associated with the s smallest eigenvalues $\hat{\lambda}_1, ..., \hat{\lambda}_s$ provide estimates for the matrix $(\delta_0', \delta_1', \ldots, \delta_m')'$.

Alternatively, an efficient estimation of elements of $\delta(L)$, including the standard errors, is obtained by FIML in a model with s pseudo structural equations where the right hand side variables is given only by the $(\Delta y_{t-1}', .., \Delta y_{t-m}')'$ and additional $(n - s)$ unrestricted equations such that

$$\begin{pmatrix} I_s & \underline{\delta}_0' \\ 0_{(n-s)\times s} & I_{n-s} \end{pmatrix} \Delta y_t = \begin{pmatrix} 0_{s\times r} & \delta_1' & \ldots & \delta_m' & 0_{s\times n} & \ldots & 0_{s\times n} \\ \tilde{\alpha} & \tilde{\Gamma}_1 & \ldots & \tilde{\Gamma}_m & \tilde{\Gamma}_{m+1} & \ldots & \tilde{\Gamma}_{p-1} \end{pmatrix} \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} + \upsilon_t, \tag{9}$$

where, without loss of generality, the contemporaneous cofeature matrix has been normalized such that $\delta_0' = (I_s, \underline{\delta}_0')$.[2]

2.2 The Coincident and Leading Indexes

We are now in the position to present our notion of coincident and leading indexes.

[2] System (9) also shows how to obtain the number of degrees of freedom d. Indeed, the unrestricted VECM needs $n \times (r + n(p - 1))$ parameters. The restricted model (9) consists in estimating $(n - s) \times (r + n(p - 1))$ $+ s \times m \times n + (sn - s^2)$ mean parameters. There are thus $s \times (n(p - 2 - m) + r + s)$ restrictions under the PSCCF null hypothesis.

Definition 2 ***CI&LI.*** *CI_t and LI_t are respectively the composite coincident and leading indexes iff*

$$E(\Delta CI_{t+1}|\Omega_t) = E(\Delta CI_{t+1}|\Delta LI_t), \qquad (10)$$

where CI_t is a linear combinations of series y_t, and LI_t is a linear combinations of series $(y_t', .., y_{t-m+1}')'$.

Suppose that there exists a single PSCCF(m) among the series Δy_t. In view of Definition 1, we have that

$$E(\delta_0' \Delta y_{t+1}|\Omega_t) = \underline{\delta}(L)' \Delta y_t,$$

where $\underline{\delta}(L) = \sum_1^m \delta_i L^{i-1}$. Consequently the coincident and leading indexes are simply given by

$$CI_t = \delta_0' y_t,$$

and

$$LI_t = \underline{\delta}(L)' y_t.$$

It is easy to see that the reverse implication holds as well, i.e. if there exists a pair of CI&LI according to Definition 2 then series Δy_t have at least one PSCCF(m). These results, along with Equation (6), imply that our CI&LI have the following important property:

Proposition 3 *Let us define the detrended CI&LI respectively as $CI_t^\xi \equiv \delta_0' \xi_t$ and $LI_t^\xi \equiv \underline{\delta}(L)' \xi_t$. Then we have*

$$E(CI_{t+h}^\xi / \Omega_t) = E(LI_{t+h-1}^\xi / \Omega_t), \quad h \geq m.$$

The above proposition tells us the expected cyclical movements of the LI lead those of the CI when the forecast horizon is not less than the PSCCF order. Hence, the case of the PSCCF(1) is particularly attractive for our CI&LI building. In the rest of the paper we will focus on such particular case.

A relevant feature of our approach is that there is no need of *a priori* classification of y_t in leading or coincident indicators. However, one may wish to test for linear hypotheses on the coefficient matrices δ_0 and δ_1 and consequently to develop a restricted estimator of the CI&LI coefficients. In particular, let us assume that the vector of n time series may be partitioned into two subvectors such that $y_t = (z_t', x_t')'$. The first n_1 series in z_t are the main variables of interest for the analysis whereas the remaining $n_2 = n - n_1$ series x_t should Granger-cause the reference series z_t. Then a natural hypothesis to test is that the CI is formed by these reference series only.

Alike Johansen (1995) in cointegration analysis, we express these linear restrictions as follows

$$\delta \equiv (\delta_0', \delta_1')' = \underbrace{H}_{n\times m}\underbrace{\varphi}_{m\times s} \equiv \begin{pmatrix} \underbrace{H_{11}}_{n\times m_1} & \underbrace{H_{12}}_{n\times m_2} \\ \underbrace{H_{21}}_{n\times m_1} & \underbrace{H_{22}}_{n\times m_2} \end{pmatrix} \begin{pmatrix} \underbrace{\varphi_0}_{m_1\times s} \\ \underbrace{\varphi_1}_{m_2\times s} \end{pmatrix}, \tag{11}$$

where H is matrix of known elements, φ is a parameter matrix to be estimated and $m = m_1 + m_2$.

Let us a consider the illustrative example where the reference series z_t are The Conference Board coincident indicators, namely the industrial production, employment, real income, and manufacturing and trade sales.[3] Then the matrix H takes the form

$$H = \begin{bmatrix} \begin{pmatrix} I_4 \\ 0_{(n-4)\times 4} \end{pmatrix} & 0_{n\times n} \\ 0_{n\times 4} & I_n \end{bmatrix}, \tag{12}$$

which means that δ_1 is unrestricted, there are no cross restrictions between δ_0 and δ_1, and $\delta_0' = (\varphi_0', 0_{4\times(n-4)})$.

We can handle such linear restrictions by means of the following procedure. We first solve the following canonical correlation program

$$CanCor \left\{ H' \begin{pmatrix} \Delta y_t \\ -\Delta y_{t-1} \end{pmatrix}, \begin{pmatrix} \hat{\beta}' y_{t-1} \\ \Delta y_{t-1} \\ \Delta y_{t-2} \\ \vdots \\ \Delta y_{t-p+1} \end{pmatrix} \mid D_t \right\}. \tag{13}$$

[3] The complete list of individual indicators considered by The Conference Board can be found at the webpage `http://www.tcb-indicators.org/GeneralInfo/serieslist.cfm`.

Then the LR test statistic is

$$LR_1 = T\sum_{i=1}^{s} \ln\left(\frac{1-\hat{\lambda}_i}{1-\widehat{\eta}_i}\right), \qquad s = 1,\ldots,m_1, \tag{14}$$

where $\widehat{\eta}_i$ is the $i-$th smallest squared canonical correlation coming from (13) and the estimates of the parameters $(\varphi_0', \varphi_1')'$ are the eigenvectors associated with the s smallest eigenvalues $\widehat{\eta}_1, ..., \widehat{\eta}_s$. The test statistic (14) follows asymptotically a $\chi^2_{s(2n-m)}$ distribution under the null hypothesis.[4]

Notice that when $s > 1$ there is not necessarily a unique CI&LI pair. In the sequel we consider both the case where several indexes are individually identified[5] and the most usual case where a unique CI&LI pair must be constructed.

2.3 Identifying the CI&LI's

In this section we show how to identify "structural" pairs of CI&LI's by means of overidentifying restrictions. Coming back to our illustrative example, we may wish to construct a coincident index that only includes the four Conference Board coincident series. In this case, we need to test for zero canonical correlations between $(\Delta z_t', -\Delta y_{t-1}')'$ and the past of y_t.

More generally, suppose that we are willing to consider only composite indexes with weights which obey to the linear restrictions (11). Then the LR test statistic for the null hypothesis that there exist s "restricted" PSCCF vectors against the alternative that no restricted PSCCF vectors exist is given by

$$LR_2 = -T\sum_{i=1}^{s} \ln(1-\widehat{\eta}_i), \qquad s = 1,\ldots,m_1. \tag{15}$$

Under the null hypothesis the test statistic (15) is asymptotically distributed as a $\chi^2_{(d)}$ with $d = s \times (n(p-1) + r + s - m)$.

[4] Notice that we could also consider a more elaborated example where the reference series z_t are variables related to the real sector of the economy and series x_t are financial variables. Then we may wish to construct distinct CI&LI for the different sectors. Moreover, we may require that the real variables do not enter in the LI of the financial sector. This kind of restrictions can be handled by a switching procedure similar as those used to test for separation in cointegration (Konishi and Granger, 1992) and in common cyclical features (Hecq et al., 2001).

[5] For example, Banerji and Hiris (2001) build multidimensional coincident and leading indicators for employment, activity, inflation.

Remark 4 *When $n_1 = 1$, namely we have a single reference series*[6]*, and we want to build a leading index for such a variable, equation (10) implies that there must exist a polynomial vector $\delta(L)$ such that*

$$\delta_0 = (1, \underbrace{0'}_{1\times(n-1)})', \text{ and } \delta_1 \neq 0. \tag{16}$$

In this case, there exists a unique LI of z_t which is of course given by $LI_t = \delta_1' y_t$. Notice that (16) is equivalent to the following restrictions

$$(1, 0') \begin{pmatrix} \underbrace{\alpha_1'}_{1\times r} \\ \underbrace{\alpha_2'}_{(n-1)\times r} \end{pmatrix} = 0 \Leftrightarrow \alpha_1 = 0, \tag{17}$$

and

$$(1, 0') \begin{pmatrix} \underbrace{\Gamma_1(L)'}_{1\times n} \\ \underbrace{\Gamma_2(L)'}_{(n-1)\times n} \end{pmatrix} = \delta(L)' \Leftrightarrow \Gamma_1(L)' = (1, 0') - \delta_1' L. \tag{18}$$

Statistical inference on the existence of such LI is easily performed. Indeed, the restrictions (17) and (18) can be checked by F-tests on the first equation of the VECM and the parameters δ_1 can be estimated by FIML in system (9).

2.4 Building the Optimal CI&LI

In this section we show how to combine several CI&LI's in order to extract the most relevant pair for forecasting purposes. More precisely, we propose the following notion of optimal coincident and leading composite indexes.

Definition 5 ***Optimal CI & LI.*** *$CI_t^* \equiv \xi^{*\prime} CI_t$ and $LI_t^* \equiv \xi^{*\prime} LI_t$ are respectively the optimal composite coincident and leading indexes iff*

$$\xi^* = \underset{(\xi)}{\arg\min} \left\{ \frac{\xi' V(e_t) \xi}{\xi' V(\Delta CI_t) \xi} \right\}, \tag{19}$$

[6] *For instance, the OECD builds its CI&LI's assuming that the reference series is the industrial production index only, see the WEB page* `http://www1.oecd.org/std/li1.htm` *for details.*

where ξ is a generic s-vector, $e_t \equiv \Delta CI_t - E(\Delta CI_t|\Delta LI_{t-1})$ and $V(\cdot)$ is the covariance matrix of the process in argument.

Notice that when several PSCCF vectors may exist (i.e., $s > 1$), condition (19) requires that ΔCI_t^* and ΔLI_{t-1}^* are the most correlated among all the linear combinations of Δy_t and Δy_{t-1} which satisfy equation (10). Based on a standard result from canonical correlation theory, equation (19) is solved by $\xi^* = [V(\Delta CI_t)]^{-1/2}\zeta_1$, where ζ_1 is the eigenvector associated to the smallest eigenvalue of the matrix

$$[V(\Delta CI_t)]^{-1/2}V(e_t)[V(\Delta CI_t)]^{-1/2}. \tag{20}$$

Hence, the optimal CI weights are given by $\delta_0^* = \delta_0\xi^*$ and the optimal LI weights are given by $\delta_1^* = \delta_1\xi^*$. We summarize the above results in the following proposition.

Proposition 6 ***Construction of the optimal CI&LI.*** *Suppose that there exists $n \times s$ PSCCF matrix $\delta(L) = \delta_0 - \delta_1 L$ with $\delta_1 \neq 0$. In this case, the optimal CI and LI are respectively given by $CI_t^* = \delta_0^{*\prime}y_t$ and $LI_t^* = \delta_1^{*\prime}y_t$, where $\delta_0^* = \delta_0\xi^*$, $\delta_1^* = \delta_1\xi^*$, $\xi^* = [V(\Delta CI_t)]^{-1/2}\zeta_1$, and ζ_1 is the eigenvector associated to the smallest eigenvalue of the matrix (20).*

The optimal CI&LI weights can be estimated as follows. Compute an estimate $\widehat{\delta}(L)$ of the PSCCF vectors and fix $\delta(L) = \widehat{\delta}(L)$. Then obtain $\widehat{\xi}^*$ by solving equation (19) where $V(\Delta CI_t)$ and $V(e_t)$ are respectively substituted with the sample covariance matrices of $\delta_0'\Delta y_t$ and $\delta(L)'\Delta y_t$. Finally, the point estimates of δ_0^* and δ_1^* are respectively given by $\widehat{\delta}_0^* = \widehat{\delta}_0\widehat{\xi}^*$ and $\widehat{\delta}_1^* = \widehat{\delta}_1\widehat{\xi}^*$.

Linear restrictions on δ_0^* and δ_1^* may be tested by a linear switching algorithm similar as the one proposed by Johansen (1995) in cointegration analysis. In particular, suppose that one wishes to consider the following system of hypothesis:

$$\mathrm{H}_0 : \delta^* \equiv (\delta_0^{*\prime}, \delta_1^{*\prime})' = H^*\varphi^* \equiv (\underbrace{H_0^{*\prime}}_{m\times n}, \underbrace{H_1^{*\prime}}_{m\times n})'\varphi^* \equiv H^*\varphi^* \text{ vs } \mathrm{H}_1 : \delta^* \text{ is unrestricted},$$

where H^* is a matrix of known elements and φ^* is a $m \times 1$ parameter matrix.

Let's write $\delta^\# = (\delta_0', \delta_1')'\xi^\#$, where $\xi^\# = (\delta_0'\Sigma_{\Delta y}\delta_0)^{-1/2}\overline{\zeta}_1$, and $\overline{\zeta}_1$ is the matrix of the $(s-1)$ eigenvectors associated to the $(s-1)$ largest eigenvalues of the matrix (20). Thus the iterative procedure goes as follows

1. Estimate $\delta^{\#}$ unrestricted by $\widehat{\delta}^{\#} = (\widehat{\delta}_0', \widehat{\delta}_1')'\widehat{\xi}^{\#}$.

2. For fixed $\delta^{\#} = \widehat{\delta}^{\#}$, obtain $\widehat{\varphi}$ as the eigenvector associated with the smallest eigenvalue coming from the solution of

$$CanCor\left\{H^{*\prime}\begin{pmatrix}\Delta y_t\\ -\Delta y_{t-1}\end{pmatrix}, \begin{pmatrix}\hat{\beta}' y_{t-1}\\ \Delta y_{t-1}\\ \Delta y_{t-2}\\ \vdots\\ \Delta y_{t-p+1}\end{pmatrix} \mid \delta^{\#\prime}\begin{pmatrix}\Delta y_t\\ -\Delta y_{t-1}\end{pmatrix}, D_t\right\}. \tag{21}$$

3. For fixed $\delta^* = H^*\widehat{\varphi}^*$, obtain $\widehat{\delta}^{\#} = \delta_\perp^* \widehat{\phi}_{(s-1)}$, where $\widehat{\phi}_{(s-1)}$ are the eigenvectors associated with the $(s-1)$ smallest eigenvalues coming from the solution of

$$CanCor\left\{\delta_\perp^{*\prime}\begin{pmatrix}\Delta y_t\\ -\Delta y_{t-1}\end{pmatrix}, \begin{pmatrix}\hat{\beta}' y_{t-1}\\ \Delta y_{t-1}\\ \Delta y_{t-2}\\ \vdots\\ \Delta y_{t-p+1}\end{pmatrix} \mid \delta^{*\prime}\begin{pmatrix}\Delta y_t\\ -\Delta y_{t-1}\end{pmatrix}, D_t\right\}. \tag{22}$$

4. Continue with 2. and 3. until numerical convergence.

The LR test statistic is

$$LR_3 = T\left[\sum_{i=1}^{s}\ln(1-\hat{\lambda}_i) - \ln(1-\widehat{\rho}_1) - \sum_{i=1}^{s-1}\ln(1-\widehat{v}_i)\right], \qquad s = 1\ldots n-r, \tag{23}$$

where $\widehat{\rho}_i$ and $\widehat{v}_i$ are the $i-$th smallest squared canonical correlations respectively coming from (21) and (22). The test statistic (23) follows asymptotically a $\chi^2_{(2n-m)}$ distribution.

3 The Long Leading Indicator

We have so far focused on building CI&LI's when the time delay is one period only. However, it is often desirable to anticipate the state of economic activity with a larger advance. Hence, the properties of the composite indexes must be evaluated also when the forecast horizon is larger than one. By construction of the CI&LI we get

$$\Delta CI_t = \Delta LI_{t-1} + e_t, \tag{24}$$

where $e_t = \delta_0' \varepsilon_t$.

Equation (24) implies in turn that

$$E(\Delta CI_{t+h}|\Omega_t) = E(\Delta LI_{t+h-1}|\Omega_t), \tag{25}$$

for $h \geq 2$. Hence, the h-step ahead forecasts of the first differences of CI are given by the $(h-1)$-step ahead forecasts of the first differences of LI.

Notice that the left hand side of equation (25) is generally not a function of ΔLI_t only. For instance, for $h = 2$ we get

$$E(\Delta LI_{t+1}|\Omega_t) = \delta_1'(\alpha\beta' y_t + \sum_{i=0}^{p-2} \Gamma_i \Delta y_{t-i}),$$

which is generally different from $E(\Delta LI_{t+1}|\Delta LI_t)$.

Based on equation (25), we define the h-step ahead leading index LI_t^h as follows

$$LI_t^h = \delta_1' E(y_{t+h-1}|\Omega_t). \tag{26}$$

In order to build such h-step ahead leading index we may follow two different approaches.

The first approach requires to derive the $(h-1)$-step ahead forecasts of series y_t from the VECM with the PSCCF and combine them with the estimated LI weights $\widehat{\delta}_1$. A possible way to incorporate the PSCCF restrictions within the VECM is to rely on the following common factor representation

$$(I_n - \Gamma_1^* L)\Delta y_t = \Lambda F_{t-1} + \varepsilon_t, \tag{27}$$

where $\Gamma_1^* = \delta_0(\delta_0'\delta_0)^{-1}\delta_0'\Gamma_1$, Λ is a full-rank $n \times (n-s)$ matrix such that $\delta_0'\Lambda = 0$,

$$F_{t-1} = \widetilde{\alpha}\beta' y_{t-1} + \sum_{i=1}^{p-1} \widetilde{\Gamma}_i' \Delta y_{t-i},$$

$\widetilde{\alpha}$ is a $(n-s)\times r$ matrix, and $\widetilde{\Gamma}_i$ is $n\times(n-s)$ matrix for $i=1,2,,...,p-1$. Efficient estimates of the parameters $[\widetilde{\alpha},\widetilde{\Gamma}_1',\widetilde{\Gamma}_2',...,\widetilde{\Gamma}_{p-1}']$ are provided by the canonical variates coefficients of $(y_{t-1}'\beta,\Delta y_{t-1}',...,\Delta y_{t-p+1}')'$ associated to the $(n-s)$ largest eigenvalues $\hat{\lambda}_{s+1},...,\hat{\lambda}_n$ coming from the solution of (7) with $m=1$. Finally, the remaining parameters of model (27) are easily estimated by a regression of Δy_t on $(\Delta y_{t-1}',F_{t-1}')'$.

The second approach consists in estimating LI_t^h conditional on the estimated parameters of LI_t by a single-equation method. In particular, for fixed $\delta_1=\widehat{\delta}_1$, we should estimate the equation

$$\Delta LI_{t+h-1}=\gamma_0^{h\prime}\beta' y_t+\sum_{i=0}^{p-2}\gamma_i^{h\prime}\Delta y_{t-i}+e_{t+h-1}^h, \tag{28}$$

where γ_0^h is a r-vector, γ_i^h is a n-vector for $i=0,1,...,p-2$, and e_t^h is a MA$(h-2)$ error. Clearly, LI_t^h is then obtained by subtracting the residuals $\widehat{e}_{t+h-1}^h$ to ΔLI_{t+h-1}.

The advantage of the second approach is that additional restrictions may easily be tested and imposed on equation (28). However, one should keep in mind that statistical inference on (28) is conditional on the estimated LI_t's parameters and hence the sample variability of $\widehat{\delta}_1$ is ignored.

An interesting question to be posed is if one can build an optimal CI&LI pair such that LI_t^* is a valid leading indicator for any forecast horizon of CI_t^*. Such CI&LI should satisfy, along with condition (19), the following equation

$$E(\Delta CI_{t+h}^*|\Omega_t)=E(\Delta LI_{t+h-1}^*|\Delta LI_t^*), \tag{29}$$

for any $h\geq 1$.

In view of equation (25) and keeping in mind that ΔCI_t^* and ΔLI_t^* are stationary ARMA processes, we see that equation (29) is satisfied when

$$\Delta LI_t^*=\rho\Delta LI_{t-1}^*+\nu_t, \tag{30}$$

where $\rho\neq 0$, $|\rho|<1$, and $\nu_t=\delta_1^{*\prime}\varepsilon_t$.

Equation (30) implies that the optimal leading index is an ARIMA(1,1,0) process. But we need a stronger requirement that the error term of this ARIMA process is an innovation with respect Ω_{t-1}. In the terminology of Granger and Yoon (2001), the optimal LI must be a self-generating variable.

By comparing equation (24) with equation (30) and in view of Proposition 5, we conclude that condition (29) holds when $\delta_1^* = \rho\delta_0^*$. This non-linear restriction can be tested and possibly imposed in estimation by means of a grid search of the likelihood function over different values of ρ.

4 Coincident and Leading Indexes for the US Economy

This section illustrates the use of the PSCCF framework for modeling coincident and leading indicators for the US economy. This analysis aims at extracting historical components for these variables. Consequently, we have left the crucial role of forecasting the business cycle and/or its turning points for further investigations. So we have not considered the important issue of how promptly the data are published. Indeed, it is well known that while some economic indicators are published a few days after the end of the month, one can wait several month to obtain (provisional) information on other series (see Moore 1991). We have also avoided the issue of structural breaks and parameter instability by assuming that the relations we obtain are constant on the whole period.

4.1 Variable Definitions and Description

We analyze the US business cycle using the monthly Business Cycle Indicators (BCI) revised composite indexes database (March 2000) created by The Conference Board (TCB). The data span the period 1959:01 to 1999:11, i.e. 491 monthly observations. Table 1 reports the variables we use together with their BCI code as well as their label in this paper. We want to compare our indexes with the TCB ones. Consequently we start with the partition $z_t = (z_{1t}, \ldots, z_{4t})$ and $x_t = (x_{1t}, \ldots, x_{10t})$. The third column reports the conclusions of the ADF unit root tests. Only the vendor performance (x_{4t}), the interest rate spread (x_{9t}) and the building permits (x_{6t}) series appear to be I(0). The fourth column reports the transformation we apply to the data. We take another route than the one chosen by Stock and Watson (1991, 1993) and Issler and Vahid (2002) for instance. In their papers the authors generally take the first differences of non-stationary variables because they do not find cointegration among the four TCB coincident indicators. In this paper instead, we test for cointegration in a larger information set which also includes some leading indicators. Since a few of these leading indicators are I(0), we prefer to integrate them, namely we take their cumulative sum in the analysis (see also Rahbek and Mosconi, 1999). This operation allows to include in the first differences of the CI&LI also the I(0) variables that otherwise would have been annihilated by Condition 1 of Definition 1. Moreover, the volatility of these series has been adjusted as TCB suggests, that is all the first differences of these series have unitary standard deviations and are now all expressed in comparable scale.

Insert Table 1

Stock and Watson (1991, 1993) impose a single dynamic common factor for summarizing the information contained in the past. Because we want to extract a CI&LI pair without doing such assumption, our approach is prone to a dimensionality problem. Consequently we have retained the following step-wise procedure based on the minimization of an information criterion such as the SBC. We start with the four variables $z_t = (z_{1t}, \ldots, z_{4t})$ and we select the VAR order that minimizes the SBC from 0 up to $p^{\max}$. Then, we add separately in the *rhs* the lags of each of the ten TCB leading indicators and we estimate all the VARX models with order from 0 up to $p^{\max}$. Finally, we compare the smallest SBC of these $10 \times p^{\max}$ VARX models with the SBC of the previously selected VAR model. If the value of the SBC is smallest for the VAR model, we keep only the reference series in the analysis. Otherwise, each of the nine remaining TCB indicators is included as an additional exogenous variable and the SBC is computed for all the $9 \times p^{\max}$ VARX models. The procedure stops when it is not possible to find a better VARX model according to the SBC. For our sample, the "elected" series are successively the building permits, average weekly hours, interest rate spread, and vendor performance, that is $y_t = (z_{1t}, z_{2t}, z_{3t}, z_{4t}, x_{1t}, x_{4t}, x_{6t}, x_{9t})$.

4.2 Cointegrating and Common Feature Relationships

Table 2 reports both the asymptotic (*Trace)* and the small sample corrected version (*Trace*^{ss}) of the Johansen trace statistics in a VAR(3). We do not reject the presence of three cointegrating vectors and then five common trends. A "*" denotes a rejection of the null hypothesis of no cointegration with a 5% significance level. A graphical inspection of the cointegrating vector confirms the outcome of the formal analysis.

Insert Tables 2 and 3

We fix at three the number of cointegrating vectors and we pursue the PSCCF analysis. Table 3 reports the $p-values$ associated with the LR_0 test statistics (8). It emerges that we cannot exclude the presence of $s=3$ PSCCF vectors but we would not retain any SCCF vectors.

4.3 Testing for the CI and LI Restrictions

The next step is to analyze appropriate zero restrictions in order to fit the CI and LI requirements. We first test whether there exists a PSCCF vector such that the CI includes the four TCB coincident series only. We use the test statistic LR_2 in (15) with the same H matrix of restrictions as in equation (12). One cannot reject the presence of a single restricted PSCCF vector with a $p-value=0.085$. It is also possible to evaluate additional restrictions on the leading indicator coefficients. As a result of a general to specific testing procedure, we cannot reject the null hypothesis that employment (z_{1t}) and sales (z_{4t}) are purely coincident indicators. In particular, we get a $p-value=0.096$ associated with the test statistic LR_2 for these joint restrictions on both the CI and the LI. Table 4 reports, for both the optimal and the restricted CI&LI, the associated coefficients[7] and the correlations between the first differences of the CI&LI and the four reference series. We call "Optimal" the CI&LI's obtained by the optimal combination given in Definition 5. We see that the changes of the reference series exhibit stronger positive correlations with the changes of the restricted CI&LI than with those of the optimal CI&LI. The correlation between the first differences of the optimal CI&LI is 0.94 whereas the correlation between the first differences of the restricted CI&LI reduces to 0.57.[8]

Insert Table 4

So far, we have obtained the coincident and the one month leading indexes. In several situations it can be interesting to foresee the economic fluctuations several months in advance. Table 5 reports the estimation of such a six month Long Leading Index (LLI_t) obtained using Equation (28). The last column shows the estimation results for which we have omitted insignificant variables at the 20% level. The correlation coefficients between the restricted long leading

[7] Notice that such coefficients are normalized such that the sum of the absolute value of the CI weights is equal to one.

[8] Such difference between the correlation coefficients is likely due to the autocorrelation of the four TCB leading indicators and it is not *per se* a reason to prefer the unrestricted model.

index ΔLLI_{t-6} and the first difference of the four coincident indicators are now respectively 0.42, 0.26, 0.19 and 0.13 while the correlation between ΔLLI_{t-6} and ΔCI_t is 0.31.

INSERT TABLE 5

Figure 1 plots the first differences of the restricted CI&LI as well as the 6-step ahead LI. To highlight the relationships with the phases of the business cycle, we shade the contraction periods according to the NBER chronology. It emerges that these indexes are quite accordant with the NBER business cycle chronology. Finally, we wish to compare the coincident index given by the PSCCF approach with two other composite indexes, namely the Stock-Watson and the TCB composite coincident indicators. These series are graphed in Figure 2 (1990.01 = 100). It is apparent that, despite some dissimilarities in the trend components, the three indexes provide a rather similar picture of the business cycle and its turning points. However, an optimal LI is available for the CI based on PSCCF.

INSERT FIGURES 1 AND 2

5 Conclusions

This paper has presented a new method to build a CI and a LI from a set of cointegrated time series y_t. Based on the notion of PSCCF (Cubadda and Hecq, 2001), the CI and LI are respectively obtained as linear combinations of y_t and y_{t-1} such that the changes of the LI are the best linear predictors of the changes of the CI. The proposed methodology covers also additional aspects of composite indicators building such as testing on the CI&LI weights and the construction of long leading indicators. Finally, concepts and methods have been illustrated by an empirical application with the US business cycle indicators.

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BCI code	Variables	DF	Transf.
	Potential Coincident Indicators		
BCI041	z_{1t} : Employees on nonagricultural payrolls (thous)	I(1)	log level
BCI051	z_{2t} : Personal income less transfer payments (Chain. Bil 96$, SAAR)	I(1)	log level
BCI047	z_{3t} : Industrial Production: Total Index (1992=100, SA)	I(1)	log level
BCI057	z_{4t} : Manufacturing & Trade Sale (Mil$, 96 Chained $)	I(1)	log level
	Potential Leading Indicators		
BCI001	x_{1t} : Average weekly hours, (hours)	I(1)	log level
BCI005	x_{2t} : Average weekly initial claims, unemploy. insurance (thous)	I(1)	log level
BCI008	x_{3t} : Mfrs' new orders, cons. good and materials (mil.chain. 1996$)	I(1)	log level
BCI032	x_{4t} : Vendor performance, slower deliveries diffusion index (%)	I(0)	Σ level
BCI027	x_{5t} : Mfrs' new orders, nondefense capital goods (mil. chain. 1996$)	I(1)	log level
BCI029	x_{6t} : Bulding permits for new private housing units (thous.)	I(0)	Σ log level
BCI019	x_{7t} : Index of stock prices, 500 common stocks, NSA (1941-1973=10)	I(1)	log level
BCI106	x_{8t} : Money supply M2 (bil. chain 1996 $)	I(1)	log level
BCI129	x_{9t} : Interest rate spread, 10-year Treasury bond less federal funds	I(0)	Σ level
BCI083	x_{10t} : Univ. of Michigan Index of Cons. Expect. (1966:I=100,NSA)	I(1)	level

Table 1: Coincident and Leading Variables

	$\widehat{\lambda}$	$VAR(p=3)$,Trace	$VAR(p=3)$,Trace^{ss}
$r=0$	0.221	293.89*	279.4*
$r \leq 1$	0.111	171.62*	163.2*
$r \leq 2$	0.088	113.81*	108.2*
$r \leq 3$	0.068	68.44	65.08
$r \leq 4$	0.036	33.58	31.93
$r \leq 5$	0.017	15.35	14.60
$r \leq 6$	0.013	6.54	6.22
$r \leq 7$	0.000	0.11	0.11

Table 2: Cointegration tests

	$\widehat{\lambda}$	PSCCF	$\widehat{\lambda}$	SCCF
$s \geq 1$	0.003	0.848	0.061	0.002
$s \geq 2$	0.007	0.901	0.113	< 0.001
$s \geq 3$	0.029	0.377	0.189	< 0.001
$s \geq 4$	0.049	0.028	0.268	< 0.001
$s \geq 5$	0.093	< 0.001	0.414	< 0.001

Table 3: p-values of PSCCF and SCCF test statistics

	Optimal		Restrict.	
	ΔCI_t^*	ΔLI_{t-1}^*	ΔCI_t	ΔLI_{t-1}
Δz_{1t}	0.3535	0.0343	0.2751	0
Δz_{2t}	-0.5538	-0.0475	0.0886	0.0821
Δz_{3t}	0.0149	0.0058	0.5400	0.2309
Δz_{4t}	0.0777	0.0352	-0.0962	0
Δx_{1t}	-0.6395	-0.0809	0	-0.0890
Δx_{4t}	1.2021	-1.2344	0	0.1258
Δx_{6t}	0.6527	-0.7171	0	0.1283
Δx_{9t}	-0.7258	0.5897	0	0.1475
Corr	$(\Delta z_{it}, \Delta CI_t^*)$	$(\Delta z_{it}, \Delta LI_{t-1}^*)$	$(\Delta z_{it}, \Delta CI_t)$	$(\Delta z_{it}, \Delta LI_{t-1})$
Δz_{1t}	0.42	0.40	0.83	0.59
Δz_{2t}	0.02	0.16	0.63	0.42
Δz_{3t}	0.17	0.15	0.94	0.47
Δz_{4t}	0.12	0.07	0.45	0.31
Corr	$(\Delta CI_t^*, \Delta LI_{t-1}^*)$		$(\Delta CI_t, \Delta LI_{t-1})$	
	0.94		0.57	

Table 4: CI and LI's - coefficients and correlations

ΔLI_t on	Unrestricted	Restricted
Variables	Coefficients	Coefficients
const.	85.613	55.319
ec_{1t-5}	-0.00101	-
ec_{2t-5}	0.00637	-
ec_{3t-5}	0.03553	0.03904
Δz_{1t-5}	0.01656	-
Δz_{2t-5}	0.01676	0.02048
Δz_{3t-5}	0.02075	0.04673
Δz_{4t-5}	-0.03579	-0.02451
Δz_{1t-6}	0.06851	0.05068
Δz_{2t-6}	-0.00598	-
Δz_{3t-6}	-0.04048	-0.03485
Δz_{4t-6}	-0.00896	-
Δx_{1t-5}	-0.03386	-0.03530
Δx_{4t-5}	-0.09427	-0.10193
Δx_{6t-5}	0.08154	0.07551
Δx_{9t-5}	0.11123	0.11121
Δx_{1t-6}	0.01001	-
Δx_{4t-6}	0.07237	0.06201
Δx_{6t-6}	0.07831	0.06729
Δx_{9t-6}	0.05983	0.06919
MA(1)	0.53565	0.552
MA(2)	0.42276	0.458
MA(3)	0.31353	0.376
MA(4)	0.25637	0.344

Table 5: 6-step ahead leading index - Equation (28)

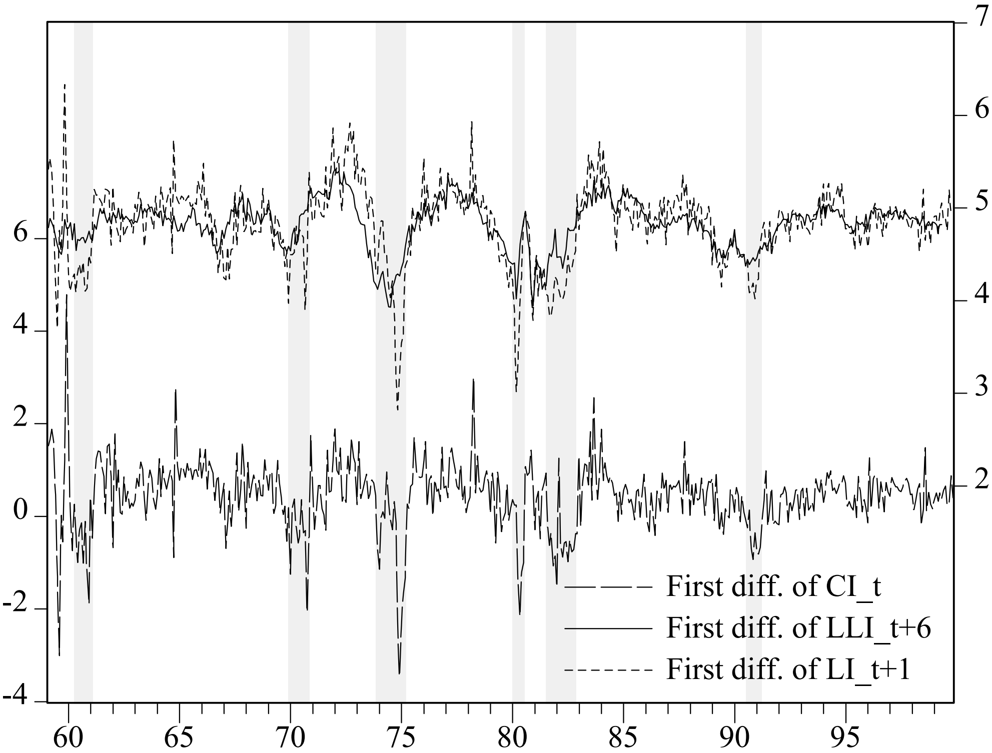

Figure 1: First differences of the restricted CI and both 1-step and 6-step ahead LI's

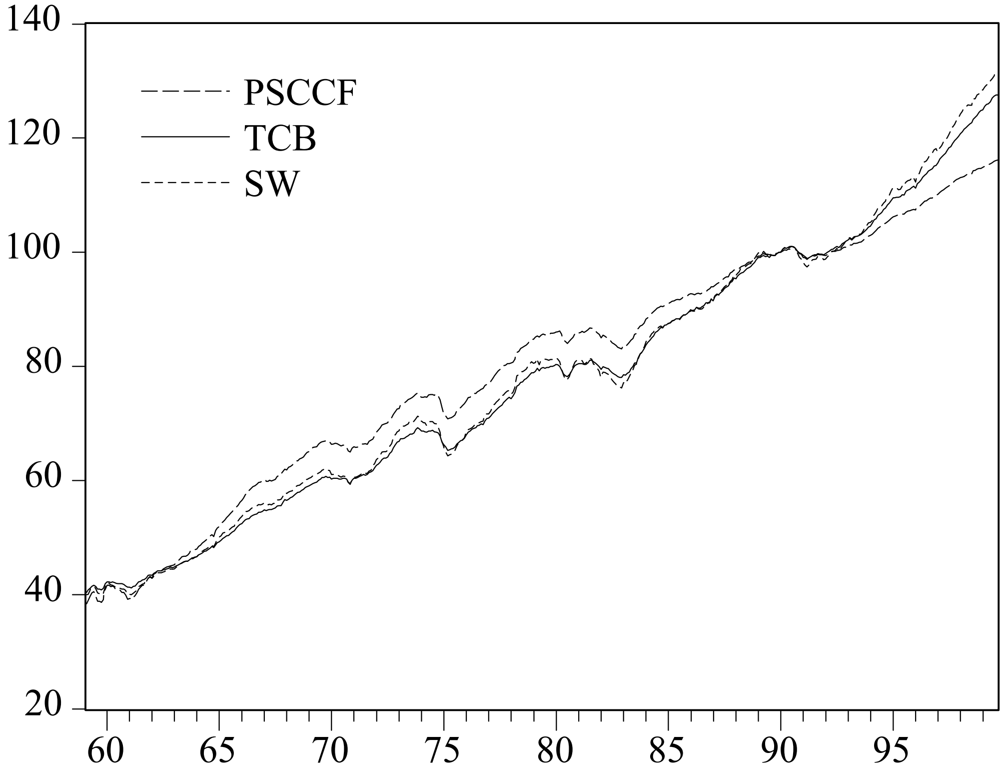

Figure 2: PSCCF (restricted), SW and TCB coincident indexes