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Common Shocks, Common Dynamics, and the International Business Cycle

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Abstract

This paper develops an econometric framework to understand whether co-movements observed in the international business cycle are the consequences of common shocks or common transmission mechanisms. Then we propose a new statistical measure of the importance of domestic and foreign shocks over the national business cycle. We show how to decompose the business cycle effects of permanent-transitory shocks into those due to their domestic and foreign components. We apply our analysis to G7 outputs.

Keywords: Common Cycles, Cointegration, Domestic-Foreign Shocks, International Business Cycles, Permanent-Transitory Decomposition.

JEL: C32

1   Introduction

The expression "international business cycle" refers to the presence of co-movements in the cyclical behavior of outputs across countries. However there exists a debate among economists and econometricians about how to measure these co-movements. In particular, the question of the importance of common shocks versus common propagation mechanisms is far from being resolved (see *inter alia* Canova and Marrinan, 1998). So is the discussion about the influence of foreign shocks over the national business cycle as well as the distinction between permanent and transitory effects of such foreign shocks. Indeed, it is crucial for economic policy purposes to understand whether national business cycles are affected by permanent technological shocks or transitory demand shocks. For instance, if demand shocks are largely responsible of fluctuations, there may be a role for aggregate Keynesian-type policies. It is also important for policy makers to know if the shocks have dominant domestic or foreign origins.

Consequently, the goal of our paper is twofold. Based on a general econometric framework, we first propose a precise empirical measure for analyzing the causes of co-movements in international business cycle. In particular that will allow us to investigate whether the observed fluctuations are due to common shocks, common propagation mechanisms or both. We exploit the low frequency co-movements coming from cointegration analysis to identify groups of shocks according to whether their effects are permanent or transitory. A serial correlation common feature analysis shows whether there exist some common dynamics, namely some common transmission mechanisms of these shocks. Imposing these restrictions will also help to estimate more accurately the responses to the shocks because redundant parameters are excluded.

Secondly, we propose a statistical measure of the importance of the domestic and foreign components of the Permanent-Transitory (henceforth, PT) shocks over the business cycle. We depart from the usual strategy that consists in extracting a unique component summarizing the worldwide effect that influences the outputs of a set of countries (see e.g. Gregory et al., 1997). Instead, the permanent [transitory] foreign shocks for each country are defined as the components of the common permanent [transitory] shocks that are independent from the national permanent [transitory] shock on that country output. Consequently, we single out a specific set of PT foreign shocks for each country as it is desirable. We then asses the importance of such foreign shocks over the national output fluctuations with a 2-8 year period. In our opinion this approach evaluates the contribution of domestic and foreign shocks to the business cycles more appropriately than the traditional impulse responses or variance decomposition.

Noticeably, our measures of the business cycle effects of the PT domestic and foreign shocks do not resort to economic theory for identifying such shocks. Indeed, if theoretical reasoning can help to disentangle the source of the various shocks within a structural VAR analysis of different

variables for a country and the rest of the world (see e.g. Kwark, 1999), it is less informative when modelling the same variable, such as output, for a larger set of countries.

The proposed approach allows us to answer a series of questions such that: 1°) Do international output measures co-move because of the existence of common shocks, common dynamics or both? 2°) What is the importance of PT foreign shocks over national business cycles, and consequently what is the degree of openness of economies? 3°) Are the business cycles mainly affected by the permanent or transitory components of domestic and foreign shocks?

This paper is organized as follows. In Section 2, we consider within a Vector Error Correction Model (VECM) a PT decomposition such that a set of cointegrated time series is separated into independent permanent and transitory components (Centoni and Cubadda, 2003). We also present the basic results relative to the common cyclical feature literature originated by Engle and Kozicki (1993) and Vahid and Engle (1993). In Section 3, we propose a statistical measure of the importance of domestic and foreign shocks over the business cycles. The role of the permanent and transitory components of domestic and foreign shocks is also investigated. Section 4 illustrates all these concepts with an emphasis to G7 output series from 1974:Q1 to 2002:Q3.

Similarly to most studies (see *inter alia* King et al, 1991), we confirm that permanent shocks are the main source of the business cycles. But in contrast to Canova and Marrinan (1998) and Mellander et al. (1992), our results suggest that foreign shocks account for a small portion of the cyclical fluctuations of the non-European G7 countries (about 13% for Japan and 25% for the US). Ahmed et al. (1993) reached a similar conclusions using a structural VAR approach. This portion is around 50% for our panel of European countries. Moreover, thanks to a finer measurement of the sources of the shocks, we deduce that the domestic component is responsible for most of the business cycle effects of transitory shocks for all the G7 countries whereas the foreign component dominates the cyclical variability that is due to permanent shocks in France, Germany and Italy.

2 Common Shocks, Common Propagation Mechanisms and Co-movements

Let X_t be a n-vector time series such that

$$A(L)X_t = \Phi D_t + \varepsilon_t, \quad t = 1,...,T \tag{1}$$

for fixed values of $X_{-p+1},...,X_0$ and where $A(L) = I_n - \sum_{i=1}^p A_i L^i$, D_t is a vector of fixed elements such a constant, a linear trend, and seasonal dummies, and ε_t are i.i.d. $N_n(0, \Omega)$ errors.

3

Let us assume that

$$|A(c)| = 0 \text{ implies that } c = 1 \text{ or } |c| > 1, \tag{2}$$

then there exist $n \times r$-matrices α and β of rank r such that $A(1) = -\alpha\beta'$. The matrix $\alpha'_{\perp}\Gamma\beta_{\perp}$ has full rank, $\alpha_{\perp}$ and $\beta_{\perp}$ are $n \times (n-r)$-matrices of rank $(n-r)$ such that $\alpha'_{\perp}\alpha = \beta'_{\perp}\beta = 0$, $\Gamma = I_n - \sum_{i=1}^{p-1}\Gamma_i$ and $\Gamma_i = -\sum_{j=i+1}^{p}A_j$ for $i = 1, 2, \ldots, p-1$. Then the process X_t is cointegrated of order (1,1), the columns of β span the cointegrating space and the elements of α are the corresponding adjustment coefficients and we can rewrite Equation (1) in the following VECM (see Johansen, 1996)

$$\Gamma(L)\Delta X_t = \Phi D_t + \alpha\beta' X_{t-1} + \varepsilon_t, \tag{3}$$

with $\Delta = (1 - L)$, and $\Gamma(L) = I_n - \sum_{i=1}^{p-1}\Gamma_i L^i$.

Series X_t admit also the following Wold representation

$$\Delta X_t = \Theta D_t + C(L)\varepsilon_t, \tag{4}$$

where $\Theta D_t = C(L)\Phi D_t$, and $C(L) = I_n + \sum_{i=1}^{\infty}C_i L^i$ is such that $\sum_{j=1}^{\infty}j|C_j| < \infty$.

Finally, we can easily obtain from Equation (4) the multivariate version of the Beveridge and Nelson (1981) representation

$$X_t = \widetilde{\Theta}\widetilde{D}_t + C(1)\sum_{j=1}^{t}\varepsilon_j + \widetilde{C}(L)\varepsilon_t, \tag{5}$$

for $\Delta\widetilde{\Theta}\widetilde{D}_t = \Theta D_t$, $\widetilde{C}(L) = \sum_{i=0}^{\infty}\widetilde{C}_i L^i$, and $\widetilde{C}_i = -\sum_{j>i}C_j$ for all i.

Since we know from the Granger representation theorem (see e.g. Johansen, 1996) that

$$C(1) = \beta_{\perp}\left(\alpha'_{\perp}\Gamma\beta_{\perp}\right)^{-1}\alpha'_{\perp}, \tag{6}$$

we see from Equation (5) that the shocks $\alpha'_{\perp}\varepsilon_t$ permanently shift the levels of series X_t. This result has motivated several authors, including Warne (1993), Gonzalo and Granger (1995), Johansen (1998), Yang (1998), and Gonzalo and Ng (2001), to conclude that the $(n-r)$ common permanent shocks are $\alpha'_{\perp}\varepsilon_t$. A natural way to identify the transitory shocks is to require that they must be independent from the permanent ones. Following this route, we get that the r common transitory shocks are $\alpha'\Omega^{-1}\varepsilon_t$. Under these assumptions, Centoni and Cubadda (2003) derived the following PT decomposition

Definition 1 (*A PT decomposition with independent components*). *Let the common permanent and transitory shocks respectively be $u_t^P = \alpha'_{\perp}\varepsilon_t$ and $u_t^T = \alpha'\Omega^{-1}\varepsilon_t$. Then the permanent and transitory components of series X_t are respectively P_t and T_t where $X_t = \widetilde{\Theta}\widetilde{D}_t + P_t + T_t$,*

$\Delta P_t = P(L)u_t^P$, $\Delta T_t = T(L)u_t^T$, and

$$P(L) = C(L)\Omega\alpha_\perp(\alpha'_\perp\Omega\alpha_\perp)^{-1}, \tag{7}$$

$$T(L) = C(L)\alpha(\alpha'\Omega^{-1}\alpha)^{-1}. \tag{8}$$

In view of equations (6) and (8), it is easy to verify that $T(1) = 0$. Hence, the shocks u_t^T have no permanent effects on series X_t as required.

Remarkably, the above decomposition is invariant to rotation of the matrices $\alpha_\perp$ and α and to non-singular linear transformations of the set of common shocks u_t^P and u_t^T. Hence, series X_t can be separated into independent PT components without using *a priori* economic theory.

After isolating the common permanent and transitory shocks by exploiting the low frequency co-movements of the data, we define what we call common transmission mechanisms. We make use of the common cyclical feature literature and in particular the notion of Serial Correlation Common Feature model (SCCF) proposed by Vahid and Engle (1993) for vector error correction models.

Definition 2 (*Serial Correlation Common Feature*). *The Series ΔX_t have s SCCF relationships iff there exists a $n \times s$ matrix δ with full column rank and such that $\delta'\Delta X_t = \delta'\Phi D_t + \delta'\varepsilon_t$. SCCF implies the following restrictions on the VECM in (3) matrix parameters: i) $\delta'\alpha = 0$ and ii) $\delta'\Gamma_i = 0, i = 1...p - 1$.*

In order to stress that the SCCF property characterizes a system with a reduced number of transmission mechanisms of the information contained in the past, we rewrite the VECM (3) in the following common factor representation

$$\Delta X_t = \Phi D_t + \delta_\perp A'W_{t-1} + \varepsilon_t \equiv \Phi D_t + \delta_\perp F_{t-1} + \varepsilon_t, \tag{9}$$

where A is a $(r+n(p-1)) \times (n-s)$ full-rank matrix, and $W_{t-1} = (X'_{t-1}\beta, \Delta X'_{t-1},..., \Delta X'_{t-p+1})'$.

Importantly enough, the main characteristic of representation (9) is that all the dynamics of the system is included in the common factors F_{t-1}. This is not generally the case in the traditional dynamic factor modeling where the idiosyncratic terms may be more cyclical than the factor itself. The price to pay with the SCCF approach is that a matrix such δ may not exist. However, we can relax the strong SCCF requirement that $\delta'\Delta X_t$ must be an innovation with respect to the past and use less stringent modeling such as the Codependence Cycle (Vahid and Engle, 1997), the Weak Form reduced rank structure (Hecq, Palm and Urban, 2000) or the Polynomial Serial Correlation Common Feature (Cubadda and Hecq, 2001) approaches. Non-linear models have also been proposed (Anderson and Vahid, 1998). Nevertheless, anticipating

the results of the empirical analysis in Section 4, we will see that SCCF is quite appropriate to restrict our statistical model of the G7 outputs. Consequently we do not present definitions of alternative common feature frameworks to save space.

The equivalence of the SCCF property to the presence of common propagation mechanism may be analyzed also through the Wold representation of series ΔX_t in (4). Indeed, we have that $\delta' C(L) = \delta'$, which means that the impulse response functions of series X_t are collinear. The SCCF has also implications for the multivariate BN representation (5). Indeed, Vahid and Engle (1993) proved that $\delta' \tilde{C}(L) = 0$, which means that the common cycles summarizing all the short-run dynamics are annihilated when premultiplying by δ'. A SCCF analysis is then a way to fully characterize the properties of a dynamic system by adding to cointegration an information concerning the common propagation mechanisms.

Maximum likelihood inference on SCCF requires to solve the following canonical correlation program,

$$
CanCor \left\{ \Delta Y_t, \left(\begin{array}{c} \hat{\beta}' Y_{t-1} \\ \Delta Y_{t-1} \\ \Delta Y_{t-2} \\ \vdots \\ \Delta Y_{t-p+1} \end{array} \right) \mid D_t \right\}, \tag{10}
$$

where $CanCor\,(Y,X \mid Z)$ denotes the partial canonical correlations between the elements of Y and X conditional on Z (netting out the effect of Z), and $\hat{\beta}$ is a superconsistent estimate of the cointegrating vectors. The likelihood ratio (LR) test for the null hypothesis that there exist at least s SCCF vectors is based on the statistic (see *inter alia* Anderson, 1984; Velu *et al.,* 1986)

$$
LR = -T \sum_{i=1}^{s} \ln(1 - \hat{\lambda}_i), s \qquad = 1,...,n \quad - r \tag{11}
$$

where $\hat{\lambda}_i$ is the $i-$th smallest squared canonical correlation coming from the solution of (10). The test statistic (11) follows asymptotically a $\chi^2_{(v)}$ distribution under the null where $v = s \times (n(p-1)+r) - s(n-s)$. Moreover, the canonical variates coefficients of ΔX_t associated with the s smallest eigenvalues $\hat{\lambda}_1,...,\hat{\lambda}_s$ provide the ML estimate of the SCCF matrix δ whereas the matrix A in equation (9) is estimated by the canonical variates coefficients of W_{t-1} associated to the $(n-s)$ largest eigenvalues $\hat{\lambda}_{s+1},...,\hat{\lambda}_n$. Finally, the matrix $\delta_\perp$ is estimated by a regression of ΔX_t on F'_{t-1}.

In addition, an efficient estimation of elements of δ, including the standard errors, is obtained, for given s, by FIML in a model with s pseudo structural equations and additional $(n-s)$

unrestricted equations such that

$$
\begin{pmatrix} I_s & \theta'_{s\times n-s} \\ 0_{(n-s)\times s} & I_{n-s} \end{pmatrix} \Delta X_t = \begin{pmatrix} 0_{s\times r} & 0_{s\times n} & 0_{s\times n} & ... & 0_{s\times n} \\ \tilde{\alpha} & \tilde{\Gamma}_1 & \tilde{\Gamma}_2 & ... & \tilde{\Gamma}_{p-1} \end{pmatrix} \begin{pmatrix} \hat{\beta}' X_{t-1} \\ \Delta X_{t-1} \\ \Delta X_{t-2} \\ \vdots \\ \Delta X_{t-p+1} \end{pmatrix} + v_t, \quad (12)
$$

where the cofeature matrix $\delta' = (I_s, \theta'_{s\times n-s})$ has been normalized and just-identified with an identity matrix on the first s elements and where $\tilde{\alpha}$ and $\tilde{\Gamma}_i$ stand for the remaining $n-s$ coefficient matrices in the VECM in (3).

Note that when we do not reject the presence of SCCF, it is also desirable to test whether the estimated relationships correspond to real linkages among variables and not to time series with idiosyncratic behavior. The latter are represented by SCCF vectors with a single element equals to 1 and the others equal to zero. These hypotheses on δ can be easily tested in System (12) with different normalizations.

3   Measuring the Business Cycle Effects of Foreign and Domestic Shocks

In this section we propose a new statistical measure of the importance of foreign and domestic shocks over the business cycle. Moreover, we show how the statistics proposed by Centoni and Cubadda (2003) may be applied to decompose the business cycle effects of PT shocks into those due to their domestic and foreign components. Hereafter we then assume that series X_t represent a set of international output measures.

Let us start by defining the pseudo-spectral density matrix of the process X_t

$$
F(\omega) = \frac{1}{2\pi} C^*(z) \Omega C^*(z^{-1})', \tag{13}
$$

where $\Delta C^*(L) = C(L)$, and $z = \exp(-i\omega)$ for $\omega \in (0, \pi]$.[1]

In order to compute $F(\omega)$ from the VECM parameters (3), we can obtain the matrices $C^*(z)$'s through the relation[2]

$$
C^*(z) = [\Gamma(z)(1-z) - \alpha\beta' z]^{-1}, \text{ for } z \neq 1.
$$

---

[1] Notice that $F(\omega)$ is unbounded at frequency zero due to the presence of unit roots at the zero frequency in series X_t.

[2] Notice that $A(z) \equiv [\Gamma(z)(1-z) - \alpha\beta' z]$ is invertible for $z \neq 1$ due to Assumption (2).

Let us now decompose each national shock in its permanent and transitory components. From the equation

$$\varepsilon_t = \varepsilon_t^P + \varepsilon_t^T,$$

where

$$\Omega\alpha_\perp(\alpha_\perp'\Omega\alpha_\perp)^{-1}\alpha_\perp'\varepsilon_t = \varepsilon_t^P,$$

$$\alpha(\alpha'\Omega^{-1}\alpha)^{-1}\alpha'\Omega^{-1}\varepsilon_t = \varepsilon_t^T,$$

we see that the jth country national shock ε_{jt}, for $j = 1, 2, ..., n$, can be separated into a permanent component ε_{jt}^P and a transitory component ε_{jt}^T.

Notice that the PT components of each national shock may affect the business cycles of other countries through two different channels. First, past PT shocks of a given country can produce their cyclical effects on foreign outputs through the propagation mechanism that is generated by the polynomial matrix $C(L)$. Second, a national permanent [transitory] shock can instantaneously influence the business cycles of other countries because elements of ε_t^P [ε_t^T] are generally dependent.

Hence, let us isolate the components of the common permanent [transitory] shocks u_t^P [u_t^T] that are explained by jth country permanent [transitory] shock ε_{jt}^P [ε_{jt}^T]. Under the assumption of normality, these components are respectively given by

$$u_{jt}^{P,D} = E(u_t^P \varepsilon_{jt}^P)[E(\varepsilon_{jt}^P)^2]^{-1}\varepsilon_{jt}^P,$$

$$u_{jt}^{T,D} = E(u_t^T \varepsilon_{jt}^T)[E(\varepsilon_{jt}^T)^2]^{-1}\varepsilon_{jt}^T,$$

where $j = 1, 2, ..., n$.[3]

We define $u_{jt}^{P,D}$ [$u_{jt}^{T,D}$] as the permanent [transitory] domestic shocks of the jth country. Consequently, we require that the permanent [transitory] foreign shocks of the jth country are the components of the shocks u_t^P [u_t^T] that are independent from jth country permanent [transitory] domestic shocks. Such permanent [transitory] foreign shocks respectively read

$$u_{jt}^{P,F} = u_t^P - E(u_t^P|\varepsilon_{jt}^P),$$

$$u_{jt}^{T,F} = u_t^T - E(u_t^T|\varepsilon_{jt}^T).$$

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[3]In the case that normality does not hold, $u_{jt}^{P,D}$ [$u_{jt}^{T,D}$] would generally be a non-linear function of the random variable ε_{jt}^P [ε_{jt}^T].

The identification of such PT domestic-foreign shocks allows us to decompose series X_t as follows

$$X_t = \widetilde{\Theta}\widetilde{D}_t + \underbrace{P_{jt}^D + P_{jt}^F}_{P_t} + \underbrace{T_{jt}^D + T_{jt}^F}_{T_t} \tag{14}$$

where

$$P_{jt}^D = P(L)u_{jt}^{P,D},$$

$$P_{jt}^F = P(L)u_{jt}^{P,F},$$

$$T_{jt}^D = T(L)u_{jt}^{T,D},$$

$$T_{jt}^F = T(L)u_{jt}^{T,F}.$$

Moreover, since each component in the RHS of equation (14) is independent from the others, we can write pseudo-spectral density matrix of the process X_t as follows

$$F(\omega) = F_{Pj}^D(\omega) + F_{Pj}^F(\omega) + F_{Tj}^D(\omega) + F_{Tj}^F(\omega) \tag{15}$$

where

$$F_{Pj}^D(\omega) = \frac{1}{2\pi}P^*(z)\Omega_j^{P,D}P^*(z^{-1})',$$

$$F_{Pj}^F(\omega) = \frac{1}{2\pi}P^*(z)\Omega_j^{P,F}P^*(z^{-1})',$$

$$F_{Tj}^D(\omega) = \frac{1}{2\pi}T^*(z)\Omega_j^{T,D}T^*(z^{-1})',$$

$$F_{Tj}^F(\omega) = \frac{1}{2\pi}T^*(z)\Omega_j^{T,F}T^*(z^{-1})',$$

and $\Omega_j^{P,D} = E(u_{jt}^{P,D}u_{jt}^{P,D\prime})$, $\Omega_j^{T,D} = E(u_{jt}^{T,D}u_{jt}^{T,D\prime})$, $\Omega_j^{P,F} = E(u_{jt}^{P,F}u_{jt}^{P,F\prime})$, $\Omega_j^{T,F} = E(u_{jt}^{T,F}u_{jt}^{T,F\prime})$.

The diagonal elements of the matrices $F_{Pj}^F(\omega)$ $[F_{Tj}^F(\omega)]$ measure the variability of outputs X_t at frequency ω that is explained by permanent [transitory] foreign shocks. Similarly, the diagonal elements of the matrices $F_{Pj}^D(\omega)$ $[F_{Tj}^D(\omega)]$ measure the variability of outputs X_t at frequency ω that is explained by permanent [transitory] domestic shocks.

We can finally propose our measures of the contribution of PT foreign [domestic] shocks to the variability of the jth country output at the business cycle frequency band.

Definition 3 (**Measures of the business cycle effects of PT foreign shocks**). *Let $I_{Pj}^F(\omega_0, \omega_1)$ $[I_{Pj}^F(\omega_0, \omega_1)]$ indicate the relative measure of the spectral mass of the jth country output at the business cycle frequency band $[\omega_0, \omega_1]$ that is explained by the jth country perma-*

nent [transitory] foreign shocks, where $0 < \omega_0 < \omega_1 \leq \pi$. Then we have

$$I_{Pj}^F(\omega_0,\omega_1) = \frac{\int\limits_{\omega_0}^{\omega_1} e_j' F_{Pj}^F(\omega)e_j d\omega}{\int\limits_{\omega_0}^{\omega_1} e_j' F(\omega)e_j d\omega},$$

$$I_{Tj}^F(\omega_0,\omega_1) = \frac{\int\limits_{\omega_0}^{\omega_1} e_j' F_{Tj}^F(\omega)e_j d\omega}{\int\limits_{\omega_0}^{\omega_1} e_j' F(\omega)e_j d\omega},$$

for $j = 1,\ldots,n$, where e_j is an n-vector with unity as its jth element and zeroes elsewhere.

Definition 4 (*Measures of the business cycle effects of PT domestic shocks*). *Let $I_{Pj}^D(\omega_0,\omega_1)$ $[I_{Tj}^D(\omega_0,\omega_1)]$ indicate the relative measure of the spectral mass of the jth country output at the business cycle frequency band $[\omega_0,\omega_1]$ that is explained by the jth country permanent [transitory] domestic shocks. Then we have*

$$I_{Pj}^D(\omega_0,\omega_1) = \frac{\int\limits_{\omega_0}^{\omega_1} e_j' F_{Pj}^D(\omega)e_j d\omega}{\int\limits_{\omega_0}^{\omega_1} e_j' F(\omega)e_j d\omega},$$

$$I_{Tj}^D(\omega_0,\omega_1) = \frac{\int\limits_{\omega_0}^{\omega_1} e_j' F_{Tj}^D(\omega)e_j d\omega}{\int\limits_{\omega_0}^{\omega_1} e_j' F(\omega)e_j d\omega},$$

for $j = 1,\ldots,n$.

Remark 5 *In view of equations (14) and (15), we see that the measures of the business cycle effects of the PT shocks proposed by Centoni and Cubadda (2003) are respectively given by*

$$I_{Pj}(\omega_0,\omega_1) = I_{Pj}^F(\omega_0,\omega_1) + I_{Pj}^D(\omega_0,\omega_1),$$

$$I_{Tj}(\omega_0,\omega_1) = I_{Tj}^F(\omega_0,\omega_1) + I_{Tj}^D(\omega_0,\omega_1),$$

for $j = 1,\ldots,n$.

Remark 6 *Based on decomposition (14), the relative contributions of foreign and domestic shocks to the variability of the jth country business cycle respectively read*

$$I_j^F(\omega_0,\omega_1) = I_{Pj}^F(\omega_0,\omega_1) + I_{Tj}^F(\omega_0,\omega_1),$$

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$$I_j^D(\omega_0,\omega_1) = I_{Pj}^D(\omega_0,\omega_1) + I_{Tj}^D(\omega_0,\omega_1),$$

for $j = 1,...,n$.

Estimation of the statistics $I_{Pj}^F(\omega_0,\omega_1), I_{Tj}^F(\omega_0,\omega_1), I_{Pj}^D(\omega_0,\omega_1)$, and $I_{Tj}^D(\omega_0,\omega_1)$ can be summarized as follows:

1. Test for cointegration and SCCF and consequently fix r and s. Estimate then a VECM, possibly under the SCCF restrictions (see Equation (9)), and derive consistent estimates of α, $\alpha_\perp$, β, $\Gamma(L)$, and Ω respectively denoted by $\widehat{\alpha}$, $\widehat{\alpha}_\perp$, $\widehat{\beta}$, $\widehat{\Gamma}(L)$, and $\widehat{\Omega}$;[4]

2. Based on the VECM residuals $\widehat{\varepsilon}_t$, construct

$$\widehat{u}_t^P = \widehat{\alpha}_\perp' \widehat{\varepsilon}_t,$$

$$\widehat{u}_t^T = \widehat{\alpha}' \widehat{\Omega}^{-1} \widehat{\varepsilon}_t,$$

$$\widehat{\varepsilon}_t^P = \widehat{\Omega}\widehat{\alpha}_\perp'(\widehat{\alpha}_\perp'\widehat{\Omega}\widehat{\alpha}_\perp)^{-1}\widehat{u}_t^P,$$

$$\widehat{\varepsilon}_t^T = \widehat{\alpha}(\widehat{\alpha}'\widehat{\Omega}^{-1}\widehat{\alpha})^{-1}\widehat{u}_t^T;$$

3. Compute $\widehat{u}_{jt}^{P,D}$ [$\widehat{u}_{jt}^{T,D}$] as the fitted values of a regression of $\widehat{u}_t^P$[$\widehat{u}_t^T$] on $\widehat{\varepsilon}_{jt}^P$ [$\widehat{\varepsilon}_{jt}^T$] and construct $\widehat{u}_{jt}^{P,F} = \widehat{u}_t^P - \widehat{u}_{jt}^{P,D}$ for $j = 1, 2, ...,n$;

4. Obtain $\widehat{\Omega}_j^{P,D}$, $\widehat{\Omega}_j^{T,D}$, and $\widehat{\Omega}_j^{P,F}$ respectively as the sample covariance matrices of the vector series $\widehat{u}_{jt}^{P,D}$, $\widehat{u}_{jt}^{T,D}$, and $\widehat{u}_{jt}^{P,F}$ for $j = 1, 2, ...,n$;

5. Construct
$$\widehat{C}^*(z_k) = [\widehat{\Gamma}(z_k)(1 - z_k) - \widehat{\alpha}\widehat{\beta}' z_k]^{-1},$$

$$\widehat{P}^*(z_k) = \widehat{C}^*(z_k)\widehat{\Omega}\widehat{\alpha}_\perp(\widehat{\alpha}_\perp'\widehat{\Omega}\widehat{\alpha}_\perp)^{-1},$$

$$\widehat{T}^*(z_k) = \widehat{C}^*(z_k)\widehat{\alpha}(\widehat{\alpha}'\widehat{\Omega}\widehat{\alpha})^{-1},$$

where $z_k = \exp(-i\omega_k)$, and $\omega_k = \omega_0(\frac{m-k}{m}) + \omega_1(\frac{k}{m})$ for $k = 0, 1, ...,m$;

6. Obtain

$$\widehat{I}_{Pj}^D(\omega_0,\omega_1) = \left[\sum_{k=0}^m e_j'\widehat{P}^*(z)\widehat{\Omega}_j^{P,D}\widehat{P}^*(z^{-1})'e_j\right]\left[\sum_{k=0}^m e_j'\widehat{C}^*(z_k)\widehat{\Omega}\widehat{C}^*(z_k^{-1})'e_j\right]^{-1},$$

---

[4]See e.g. Gonzalo and Granger (1995) on estimation of $\alpha_\perp$.

$$\widehat{I}_{Tj}^{D}(\omega_0,\omega_1) = \left[\sum_{k=0}^{m} e_j' \widehat{T}^*(z) \widehat{\Omega}_j^{T,D} \widehat{T}^*(z^{-1})' e_j\right] \left[\sum_{k=0}^{m} e_j' \widehat{C}^*(z_k) \widehat{\Omega} \widehat{C}^*(z_k^{-1})' e_j\right]^{-1},$$

$$\widehat{I}_{Pj}^{F}(\omega_0,\omega_1) = \left[\sum_{k=0}^{m} e_j' \widehat{P}^*(z) \widehat{\Omega}_j^{P,F} \widehat{P}^*(z^{-1})' e_j\right] \left[\sum_{k=0}^{m} e_j' \widehat{C}^*(z_k) \widehat{\Omega} \widehat{C}^*(z_k^{-1})' e_j\right]^{-1},$$

$$\widehat{I}_{Tj}^{F}(\omega_0,\omega_1) = 1 - \widehat{I}_{Pj}^{D}(\omega_0,\omega_1) - \widehat{I}_{Tj}^{D}(\omega_0,\omega_1) - \widehat{I}_{Pj}^{F}(\omega_0,\omega_1),$$

for $j = 1,...,n$.

The suggested measures are rather involved functions of the estimated VECM parameters and this complicates the analytical evaluation of their sample variability. Hence, we rely on a bootstrap procedure similar as the one suggested by Gonzalo and Ng (2001). First, we fix both r and s and estimate the VECM by the ML procedure.[5] Second, we obtain the residuals $\widehat{\varepsilon}_t$ by replacing the unknown parameters in equations (3) or (9) with their estimated values. Third, a new sample of data is constructed using a random sample of $\widehat{\varepsilon}_t$ with replacement and the initial estimates of the VECM parameters. Fourth, the VECM is re-estimated with the new sample and the associated estimates of the spectral measures are stored. This procedure is repeated 5000 times and the quantiles of the empirical distributions of the bootstrapped $\widehat{I}_{Pj}^{F}(\omega_0,\omega_1)$, $\widehat{I}_{Tj}^{F}(\omega_0,\omega_1)$, $\widehat{I}_{Pj}^{D}(\omega_0,\omega_1)$, and $\widehat{I}_{Tj}^{D}(\omega_0,\omega_1)$ are then used to construct confidence intervals.

4   Empirical Analysis

In this section we analyze the presence of common shocks and common transmission mechanisms in international outputs and consequently we determine the existence and the type of co-movements in the international business cycles. The measures proposed in Section 3 are applied to investigate the importance of the domestic and foreign shocks over the business cycles as well as the proportions of the permanent and transitory effects of such shocks. We consider the output of G7 countries, i.e. Canada, US, UK, Germany, Italy, France and Japan. Quarterly seasonally adjusted indexes of the Gross Domestic Product in volume (1995=100) are taken from OECD databases. Canova and Dellas (1993) among others documented that after 1973 (i.e. the first oil shock) the presence of common disturbances plays a role in accounting for international output co-movements. We then use the sample that span 1974:Q1 to 2002:Q3, namely $T = 115$ observations.

Figure 1 reports for each country taken individually, the log-level and the growth rate of the GDP.[6] It emerges that there exists a trending behavior in the log-levels of all series. We

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[5] The coefficients of β and δ matrices are estimated even though r and s are fixed.

[6] The data for Germany for the period 1974:Q1-1990:Q4 were reconstructed by using the GDP of West Germany.

	$\widehat{\lambda}VAR$	$(p=3)$	$p-values$
$r=0$	0.34	183.54	0.000
$r \leq 1$	0.33	134.17	0.002
$r \leq 2$	0.21	88.02	0.055
$r \leq 3$	0.19	60.23	0.096
$r \leq 4$	0.14	35.96	0.210
$r \leq 5$	0.09	18.54	0.315
$r \leq 6$	0.05	6.58	0.401

Table 1: Johansen's Trace Test

first test for the presence of common permanent and transitory shocks among these series by a cointegration analysis. Except for Germany, the growth rates also show a cyclical pattern whose similarity is tested through the SCCF analysis.

A VAR(3) seems to appropriately characterize the covariance structure of the data according to the AIC. In so doing, we do not reject the null of no autocorrelation in all the individual equations of the VAR. Indeed, we obtain for the vector time series

$$X_t = (\ln Can_t, \ln US_t, \ln Jap_t, \ln Fr_t, \ln Ger_t, \ln It_t, \ln UK_t)',$$

the following $p-values$ associated to the LM test statistics for fourth-order residual autocorrelation: 0.61, 0.81, 0.87, 0.29, 0.11, 0.07, 0.51.

We use the Johansen's trace statistics to determine the number of cointegrating vectors. A deterministic trend is included in the long-run to capture the differences among the average growth rates of the various national outputs. Table 1 reports the eigenvalues, the value of the asymptotic trace statistics as well as the associated $p-$values. We do not reject the presence of two cointegrating vectors. This implies that the G7 outputs are driven by five common permanent shocks and two common transitory shocks. Figure 2 displays the first six cointegrating vectors adjusted for the short-run dynamics. Visual inspection appears to confirm the outcome of the formal statistical analysis.

We fix at $r=2$ the number of cointegrating vectors and we continue with the SCCF analysis. Table 2 reports eigenvalues, the value of SCCF test statistics, their degrees of freedom as well as the $p-$values associated with both the asymptotic test in (11) and a small sample corrected version (see Hecq, 2000). It emerges that we cannot exclude the presence of four SCCF vectors. AIC also indicates $s=4$. We conclude that there are three common transmission mechanisms of the national shocks through the G7 economies.

To be complete, we pursue the analysis by testing whether some of these four SCCF vectors have a single element equals to 1 and the others equal to zero. Such trivial SCCF vectors would indicate that individual elements ΔX_t have an idiosyncratic behavior, being an innovation with
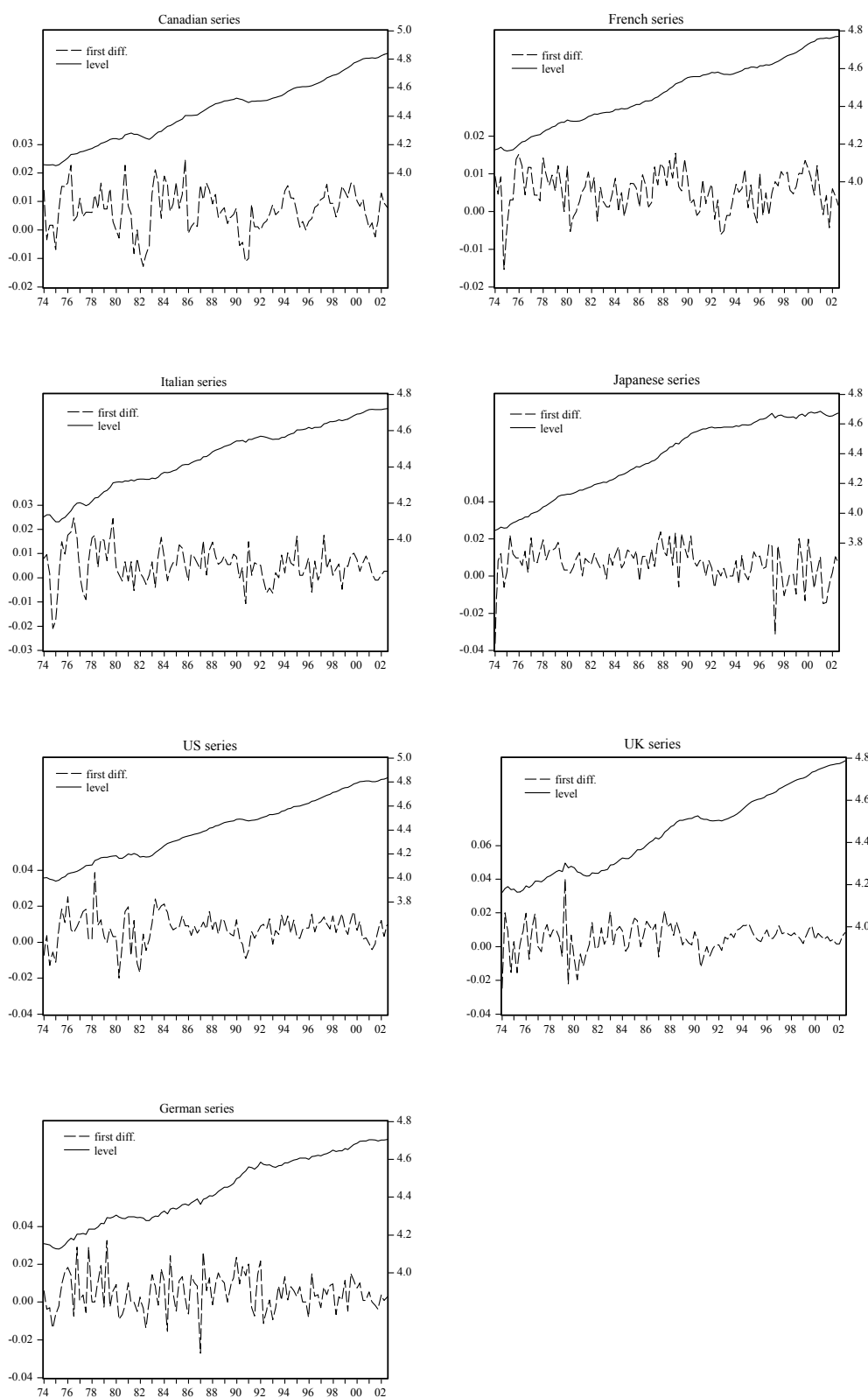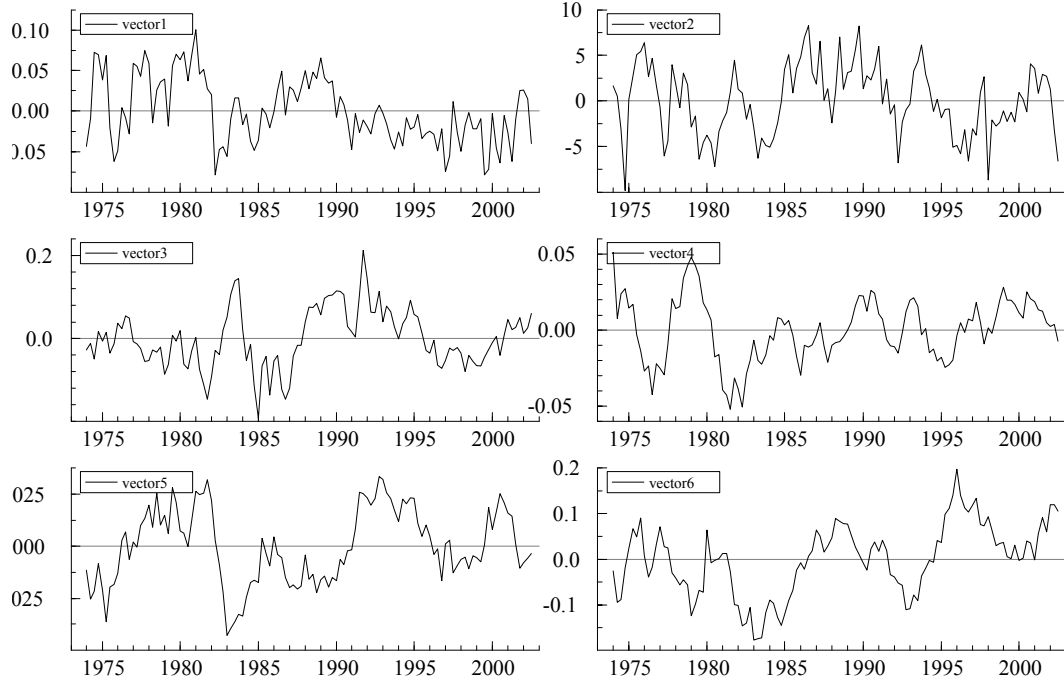
13

Figure 1: G7 output growths and log-levels

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Figure 2: Cointegration analysis: first 6 vectors

	$\widehat{\lambda} SCCFtest$		df	$p-value^p$	$p-value^{ss}$
$s \geq 1$	0.05	6.43	10	0.77	0.85
$s \geq 2$	0.12	21.37	22	0.49	0.68
$s \geq 3$	0.13	38.25	36	0.36	0.61
$s \geq 4$	0.21	65.99	52	0.09	0.30
$s \geq 5$	0.35	117.07	70	<0.001	0.01

Table 2: SCCF Test Statistics

respect to the past. To do so, we estimate by FIML System (12) for different normalizations and test for the null hypothesis that the three coefficients of each rows of $\theta'_{s \times n-s}$ are jointly equal to zero. We reject the presence of such trivial common feature vectors with $p-values$ less than 0.001 for each variables. We also reject at conventional significance levels the null hypothesis that one variable can simultaneously be excluded for the four common feature vectors. This hypothesis can also be analyzed in System (12) by FIML imposing that the four coefficients of each column of $\theta'_{s \times n-s}$ are jointly equal to zero.

In order to asses the relative importance of common PT domestic and foreign shocks over the national business cycles, we apply the measures proposed in the previous section. We compute such measures with and without imposing the SCCF restrictions in order to evaluate

		Permanent	Transitory	Total
Canada	Domestic	0.802 [0.507-0.893]	0.058 [0.017-0.090]	0.860 [0.559-0.931]
	Foreign	0.132 [0.061-0.425]	0.007 [0.002-0.029]	0.139 [0.068-0.440]
	Total	0.934 [0.894-0.971]	0.066 [0.029-0.105]	
US	Domestic	0.690 [0.421-0.821]	0.055 [0.016-0.084]	0.745 [0.465-0.861]
	Foreign	0.250 [0.135-0.520]	0.003 [0.001-0.028]	0.254 [0.138-0.534]
	Total	0.941 [0.907-0.974]	0.058 [0.025-0.092]	
Japan	Domestic	0.718 [0.436-0.827]	0.150 [0.060-0.212]	0.868 [0.580-0.918]
	Foreign	0.126 [0.072-0.412]	0.004 [0.001-0.022]	0.131 [0.081-0.418]
	Total	0.845 [0.776-0.934]	0.154 [0.065-0.223]	
France	Domestic	0.401 [0.153-0.685]	0.156 [0.051-0.204]	0.558 [0.275-0.784]
	Foreign	0.441 [0.215-0.724]	0.000 [0.000-0.001]	0.441 [0.215-0.724]
	Total	0.842 [0.795-0.947]	0.157 [0.052-0.204]	
Germany	Domestic	0.398 [0.161-0.675]	0.123 [0.037-0.176]	0.519 [0.266-0.751]
	Foreign	0.465 [0.238-0.722]	0.014 [0.002-0.023]	0.480 [0.247-0.733]
	Total	0.864 [0.811-0.954]	0.136 [0.045-0.189]	
Italy	Domestic	0.296 [0.127-0.559]	0.143 [0.045-0.196]	0.439 [0.245-0.644]
	Foreign	0.556 [0.349-0.752]	0.004 [0.001-0.010]	0.561 [0.355-0.754]
	Total	0.853 [0.798-0.951]	0.147 [0.049-0.201]	
UK	Domestic	0.484 [0.187-0.683]	0.111 [0.052-0.161]	0.595 [0.292-0.768]
	Foreign	0.346 [0.194-0.656]	0.058 [0.018-0.088]	0.404 [0.231-0.707]
	Total	0.831 [0.774-0.916]	0.169 [0.082-0.225]	

Table 3: Measures of the BC effects of Domestic-Foreign PT shocks (s=0)

the efficiency gains coming from the imposition of the common propagation mechanisms. We then estimate the VECM model fixing both $s = 0$ and $s = 4$ and derive from the estimated parameters the spectra of each output and its components at the frequencies corresponding to 8-32 quarter periods. In particular, these spectra are computed for $\omega_k = \frac{\pi}{16}\left(\frac{199-k}{199}\right) + \frac{\pi}{4}\left(\frac{k}{199}\right)$ and $k = 0, 1, ..., 199$. Table 3 and 4 report for respectively $s = 0$ and $s = 4$ the estimated measures along with in brackets the 95% bootstrapped confidence bounds.

First, the results clearly indicate the dominant role of the permanent shocks over the business cycles. Permanent shocks account for about 85% for European countries and Japan and up to 95% for the US and Canada. The imposition of the SCCF restrictions does not alter the main conclusion about these proportions but it leads to more precise estimates of the business cycle effects of PT shocks. Indeed, the relative confidence interval width of such business cycle effects[7]

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[7]Such relative confidence interval width is computed as the 95% bootstraped confidence interval width of

		Permanent	Transitory	Total
Canada	Domestic	0.826 [0.545-0.892]	0.046 [0.017-0.073]	0.872 [0.587-0.927]
	Foreign	0.112 [0.059-0.397]	0.014 [0.004-0.033]	0.127 [0.072-0.412]
	Total	0.939 [0.907-0.972]	0.061 [0.028-0.092]	
		Permanent	Transitory	Total
US	Domestic	0.614 [0.330-0.781]	0.067 [0.028-0.096]	0.682 [0.385-0.831]
	Foreign	0.312 [0.163-0.605]	0.005 [0.001-0.016]	0.317 [0.168-0.613]
	Total	0.926 [0.896-0.965]	0.073 [0.034-0.103]	
		Permanent	Transitory	Total
Japan	Domestic	0.650 [0.421-0.822]	0.161 [0.062-0.202]	0.811 [0.556-0.918]
	Foreign	0.184 [0.078-0.437]	0.004 [0.001-0.011]	0.184 [0.081-0.442]
	Total	0.834 [0.792-0.936]	0.166 [0.063-0.207]	
		Permanent	Transitory	Total
France	Domestic	0.372 [0.147-0.642]	0.130 [0.044-0.156]	0.502 [0.237-0.722]
	Foreign	0.497 [0.275-0.762]	0.000 [0.000-0.001]	0.497 [0.276-0.762]
	Total	0.869 [0.843-0.954]	0.130 [0.045-0.157]	
		Permanent	Transitory	Total
Germany	Domestic	0.414 [0.227-0.660]	0.135 [0.053-0.154]	0.549 [0.327-0.752]
	Foreign	0.445 [0.245-0.669]	0.004 [0.000-0.007]	0.450 [0.247-0.672]
	Total	0.860 [0.840-0.945]	0.139 [0.054-0.159]	
		Permanent	Transitory	Total
Italy	Domestic	0.408 [0.234-0.661]	0.101 [0.033-0.126]	0.509 [0.307-0.724]
	Foreign	0.488 [0.272-0.691]	0.002 [0.001-0.004]	0.490 [0.275-0.692]
	Total	0.897 [0.871-0.964]	0.103 [0.035-0.128]	
		Permanent	Transitory	Total
UK	Domestic	0.506 [0.254-0.720]	0.124 [0.055-0.153]	0.631 [0.360-0.801]
	Foreign	0.343 [0.182-0.611]	0.026 [0.009-0.036]	0.368 [0.198-0.638]
	Total	0.849 [0.820-0.932]	0.150 [0.068-0.179]	

Table 4: Measures of the BC effects of Domestic-Foreign PT shocks (s=4)

reduces on average of 15.44% when $s = 4$ is imposed in estimation.

Second, we turn to evaluating the importance of the domestic and foreign shocks on the different economies at the business cycle frequencies. Both fixing $s = 0$ and $s = 4$, it emerges that for the US, Japan and Canada the foreign component is small ranging between 12% and 30%. Due to their higher degree of openness, European countries are more sensitive to foreign shocks with proportions around 40% for UK and reaching 56% for Italy. Again, the main consequence of imposing the SCCF restrictions is that the relative confidence interval width of these business cycle effects reduces on average of 8.87%.

Third, for all the G7 economies, the foreign component of the business cycle is almost entirely generated by permanent shocks. These results are consistent with a possible explanation of the source of international permanent shocks where new technology is embodied in exported-imported inputs and hence countries which mainly import input goods are more exposed to foreign permanent shocks.

Finally, the domestic component clearly dominates the cyclical effects of transitory shocks, especially for European countries. This is in line with the interpretation that transitory shocks are mainly connected to national monetary and fiscal policies.

This example illustrates that the proposed methods are well suited to tackle, in an coherent and integrated setting, issues that were often analyzed independently in previous studies.

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$I_{Pj}(\omega_0,\omega_1)$ or $I_{Tj}(\omega_0,\omega_1)$ divided by the minimum of $\{\widehat{I}_{Pj}(\omega_0,\omega_1), \widehat{I}_{Tj}(\omega_0,\omega_1)\}$ for $j = 1,2,...,n$.

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