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# Change in persistence tests for panels

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# Change in persistence tests for panels*

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**Abstract**

In this paper we propose a set of new panel tests to detect changes in persistence. These statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1) or viceversa. Alternative of unknown direction is also considered. The limiting distributions of the panel tests are derived and small sample properties are investigated by Monte Carlo experiments under the hypothesis that the individual series are cross-sectionally independently distributed. These tests have a good size and power properties. Cross-sectional dependence is also considered. A procedure of de-factorizing proposed by Stock and Watson (2002) is applied. Monte Carlo analysis is conducted and the defactored panel tests show to have good size and power. The empirical results obtained from applying these tests to a panel covering 15 European countries between 1970 and 2006 suggest that inflation rate changes from I(1) to I(0) when cross-correlation is considered.

**Keywords**: Persistence, Stationarity, Panel data.
**JEL Classification**: C12, C23.

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# 1 Introduction

The recent time series literature has shown that the economic and financial data are characterized by a change in persistence between separate I(1) and I(0) regime rather than a simply I(1) or I(0) behavior. For example, Cogley and Sargent (2001) and Emery (1994), using post World War II data, argued that persistence in U.S. inflation has decreased substantially since the early 1980s. Strikingly, Emery finds that U.S. inflation in the 1980s can best be described as white noise. Further evidence of persistence change from I(1) to I(0) behavior in U.S. inflation is also reported in Kim (2000), Busetti and Taylor (2004) and Leybourne et al. (2003). Other variables for which changes in persistence have been observed include real output (e.g Taylor, 2005) and short-term interest rates (e.g. Mankiw *et al.*, 1987)).
A number of testing procedures have been developed to test against changing persistence. The most popular of these appear to be the ratio-based persistence change tests of Kim (2000), Kim *et al.* (2002), Busetti and Taylor (2004) and Harvey *et al.* (2006). These statistics test the null hypothesis that a series is a constant I(0) process against the alternative that it displays a change in persistence, either from I(0) to I(1), or viceversa. Kim (2000) and Kim *et al.* (2002) proposed residual-based ratio test against changes in persistence in a time series, focusing on the case of a shift from I(0) to I(1), at some point in the sample. Kim (2000) also discussed the possibility of I(1) to I(0) shifts but did not provide tests against such alternative. Busetti and Taylor (2004) proposed new ratio-based tests and breakpoint estimators which are consistent under I(1) to I(0) changes, and they demonstrated that the ratio-based tests which are consistent against changes from I(1) to I(0) are not consistent against changes from I(0) to I(1), and viceversa, with neither consistent against constant I(1) processes. Harvey *et al.* (2006) developed a set of new tests which are based on modified version of the ratio-base statistics of Kim (2000), Kim *et al.*(2002) and Busetti and Taylor (2004). These modifications use the variable addition approach of Vogelsang (1998), and a recent generalization due to Sayginsoy (2003), yielding tests which, by design, have the same critical values regardless of whether the process is I(0) or (near) I(1) throughout. This technique can be only used with the ratio based test of the null I(0) because other tests of the I(0) (I(1)) null are based on statistics which are divergent under constant I(1) (I(0)) processes. Hence, the null hypothesis is that of constant persistence (either a constant I(0) process or a constant I(1) process), and the alternative is that of a change in persistence. Finally, Costantini and Gutierrez (2007) consider a panel data companion of the set of recursive ADF unit root tests for single time series as proposed in Banerjee *et al.* (1992).
In this paper we propose a set of new panel tests to detect changes in persistence. These statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1) or viceversa. Alternative of unknown direction is also considered.
The paper is organized as follows. In section 2 we present new panel persistence change tests under the hypothesis of cross-section independence. Section 3 describes the panel tests under cross-section dependence hypothesis. Section 4

presents Monte Carlo simulations. In section 5 these new panel tests to a panel of 15 European inflation rate series for the period 1970.1-2006.2 are applied. Section 6 concludes.

# 2 Persistence tests without cross-section correlation

## 2.1 The model

Consider the following Gaussian unobserved components model for a sample of $N$ cross-sections observed over $T$ time periods:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + \varepsilon_{i,t}, \qquad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{1}$$

we allow for the following three cases:

- Case 1: I(0) $\rightarrow$ I(1)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [T\tau])\eta_{i,t}, \qquad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{2}$$

- Case 2: I(1) $\rightarrow$ I(0)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau T])\eta_{i,t} \qquad i = 1, \ldots, N,\ t = 1, \ldots, T. \tag{3}$$

- Case 3: unknown direction I(0) $\rightarrow$ I(1) or I(1) $\rightarrow$ I(0)

where $1(\cdot)$ is the indicator function, $d_{i,t}$ is a deterministic component, $\varepsilon_{i,t}$ and $\eta_{i,t}$ are mutually independent mean zero iid gaussian process with variance $\sigma^2_{\varepsilon i}$ and $\sigma^2_{\eta i}$. For the present, the deterministic components are taken to be the unity vector.
¿From (1)-(2), it can be easily seen that for each cross section $i$, the data generating process yields a process which is stationary up to and including time $[\tau T]$, with the change-point proportion $\tau \in (0, 1)$, but is I(1) after the break, if and only if $\sigma^2_{\eta i} > 0$. From (1)-(3), it can be easily seen that for each cross section $i$, the data generating process yields a process which is I(1) up to and including time $[\tau T]$ but it is stationary after the break, if and only if $\sigma^2_{\eta i} > 0$.
Therefore, panel test of stationarity against a shift in persistence from stationarity to a unit root or viceversa can be framed in testing the null hypothesis:

$$\mathbf{H_0} = \sigma^2_{\eta i} = 0, \ \ \forall\, i \tag{4}$$

against the alternative hypothesis

$$\mathbf{H_1} = \sigma^2_{\eta i} > 0, \text{ at least for some } i. \tag{5}$$

The following assumption plays a key role for the remaining part of the paper.

**Assumption 1** *The process* $\{\mu_{i,t}\}_{i,t=0}^{+\infty}$ *is such that for each* $i$

1. $\mathbf{E}[\mu_i] = 0$;

2. $\mathbf{E}|\mu_i|^4 < +\infty$;

3. $\{\mu_{i,t}\}_{i,t=0}^{+\infty}$ *is* $\phi$*-mixing with mixing coefficients* $\phi_m$ *such that*

$$\sum_{m=1}^{\infty} \phi_m^{1-2/\gamma} < +\infty;$$

4. *There exists the long-run variance*

$$\sigma_{\mu i}^2 = \sum_{j=0}^{\infty} \mathbf{E}[\mu_{i,j+1}\mu'_{i,1}];$$

5. *for each* $s \in (0,1)$*, we have*

$$\lim_{T\to\infty} \mathbf{V}\Big[\frac{1}{\sqrt{T}} \sum_{t=1}^{[sT]} \mu_{i,t}\Big] = s\sigma_{\mu i}^2$$

*and*

$$\lim_{T\to\infty} \mathbf{V}\Big[\frac{1}{\sqrt{T}} \sum_{t=[sT]+1}^{T} \mu_{i,t}\Big] = (1-s)\sigma_{\mu i}^2$$

.

The above conditions have been used by Phillips (1987), Phillips and Perron (1988) and Phillips and Solo (1992), among others, to prove results on the asymptotic distribution of a stochastic process. Finally, note that throughout the next sections we use sequential limits, wherein $T \to \infty$ followed by $N \to \infty$.

## 2.2 Panel ratio-based tests: I(0) $\to$ I(1)

In this section we present new panel tests to detect changes in persistence as in (2) and investigate their asymptotic behavior. We show that panel tests are standard normal distributed.
Consider the gaussian process (1)-(2). We want to test the null hypothesis $\mathbf{H_0}$ in (4) against $\mathbf{H_1}$ in (5). Let $\tilde{\varepsilon}_{i,t}$, $i = 1, \dots, N$ and $t = 1, \dots, T$, be the residuals from the regression of $y_{i,t}$ on intercept. If a structural change occurs at time $t = [\tau T]$ for $\tau \in (0,1)$, the following partial sum process can be defined:

$$\begin{cases} S_{i,t}^{(0)} = \sum_{j=1}^{t} \tilde{\varepsilon}_{i,j} \qquad t = 1, \dots, [T\tau]; \;\; i = 1, \dots, N, \\ S_{i,t}^{(1)} = \sum_{j=[T\tau]+1}^{t} \tilde{\varepsilon}_{i,j} \qquad t = [T\tau]+1, \dots, T; \;\; i = 1, \dots, N, \end{cases} \tag{6}$$

Then, we consider the following statistics-test:

$$\mathcal{K}_{T,N}(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \Big[\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} - \mu\Big], \tag{7}$$

where

$$\mu = \mathbf{E}\Big[\frac{(T-[T\tau])^{-2} \sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}\Big]$$

and

$$\sigma = \sqrt{\mathbf{V}\Big[\frac{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}\Big]}.$$

Fixed $i=1,\dots,N$, $t_1=1,\dots,[T\tau]$ and $t_2=[T\tau]+1,\dots,T$, it results $S_{i,t_1}^{(1)}$ and $S_{i,t_2}^{(0)}$ mutually independent. Therefore

$$\mu = \frac{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} \mathbf{E}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} \mathbf{E}[S_{i,t}^{(0)}(\tau)^2]} \tag{8}$$

and

$$\sigma = \sqrt{\frac{(T-[T\tau])^{-4}\sum_{t=[T\tau]+1}^{T} \mathbf{V}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-4}\sum_{t=1}^{[T\tau]} \mathbf{V}[S_{i,t}^{(0)}(\tau)^2]}} \tag{9}$$

**Theorem 2** *Suppose that Assumption 1 is true under the null hypothesis* $\mathbf{H_0}$*. Then it results*

$$\lim_{N\to+\infty}\lim_{T\to+\infty}\mathcal{K}_{T,N}(\tau)=\mathcal{K}(\tau)\sim N(0,1). \tag{10}$$

**Proof.** By Theorem 3.1 in Kim (2000), it results

$$\lim_{T\to+\infty}\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}}\frac{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} = \frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2\mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r},$$

where $\{V_i\}_{i=1}^{+\infty}$ is a sequence of standard brownian bridges that are independent and identically distributed.
Furthermore, by the hypotheses stated in Assumption 1, we have that

$$\lim_{T\to+\infty}\mathbf{E}\Big[\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}}\frac{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}\Big]=\bar{\mu},$$

$$\lim_{T\to+\infty}\mathbf{V}\Big[\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}}\frac{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}\Big]=\bar{\sigma}^2,$$

with

$$\bar{\mu}=\mathbf{E}\Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2\mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r}\Big] \tag{11}$$

and

$$\bar{\sigma}^2=\mathbf{V}\Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2\mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r}\Big] \tag{12}$$

for each $i=1,\dots,N$.
Therefore

$$\lim_{N\to+\infty}\lim_{T\to+\infty}\mathcal{K}_{T,N}(\tau)=\lim_{N\to+\infty}\frac{\sqrt{N}}{\bar{\sigma}}\cdot\frac{1}{N}\cdot\sum_{i=1}^{N}\Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2\mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r}-\bar{\mu}\Big],$$

for $i = 1, \ldots, N$.
The Central Limit Theorem guarantees that

$$\lim_{N \to +\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \Big[ \frac{(1-\tau)^{-2} \int_{\tau}^{1} V_i(r-\tau)^2 \mathrm{d}r}{N\tau^{-2} \int_0^{\tau} V_i(r)^2 \mathrm{d}r} - \bar{\mu} \Big] \sim N(0,1),$$

and the Theorem is completely proved. ■
The true value of $\tau$ is unknown. Under the situation of the true change period being unknown three transformations of the tests $\mathcal{K}_{T,N}(\tau)$ defined in (7), for testing $\mathbf{H_0}$ against $\mathbf{H_1}$ with unknown break point $[T\tau]$, can be considered.

- A maximum-Chow-type test as is considered in Davies (1977), Hawkins (1987), Kim and Siegmund (1989), and Andrews (1993) is

$$H_1(\mathcal{K}_{T,N}(\tau)) := \max_{\tau \in (0,1)} \mathcal{K}_{T,N}(\tau). \qquad (13)$$

- The mean score test proposed by Hansen (1991)

$$H_2(\mathcal{K}_{T,N}(\tau)) := \int_{\tau \in (0,1)} \mathcal{K}_{T,N}(\tau) \mathrm{d}\tau. \qquad (14)$$

- The mean-exponential test introduced by Andrews and Ploberger (1994), that is

$$H_3(\mathcal{K}_{T,N}(\tau)) := \log\Big\{ \int_{\tau \in (0,1)} \exp[\mathcal{K}_{T,N}(\tau)] \mathrm{d}\tau \Big\}. \qquad (15)$$

The asymptotic distribution of the tests defined in (13), (14) and (15) are given in the following result.

**Theorem 3** *The following propositions hold.*

*(i) It results*

$$\lim_{T \to +\infty} \lim_{N \to +\infty} H_j(\mathcal{K}_{T,N}(\tau)) = H_j(\mathcal{K}(\tau)), \qquad j = 1, 2, 3.$$

*(ii) For each $j = 1, 2, 3$, we have $H_j(\mathcal{K}(\tau)) \sim N(0,1)$.*

**Proof.**

(i) The result follows from the continuous mapping theorem and the continuity of the functionals.

(ii) Since $\mathcal{K}(\tau) \sim N(0,1)$, for each $\tau$, then $\mathcal{K}(\tau)$ is an iid continuous-time stochastic process. Therefore, we can define the random variable $\mathcal{K} \sim N(0,1)$ such that $\mathcal{K}(\tau) \equiv \mathcal{K}$, for each $\tau \in (0,1)$.
Then we have

$$H_1(\mathcal{K}(\tau)) = \max_{\tau \in (0,1)} (\mathcal{K}) = \mathcal{K} \sim N(0,1);$$

$$H_2(\mathcal{K}(\tau)) = \int_{\tau \in (0,1)} \mathcal{K} \mathrm{d}\tau = \mathcal{K} \cdot \int_{\tau \in (0,1)} \mathrm{d}\tau = \mathcal{K} \sim N(0,1);$$

$$H_3(\mathcal{K}(\tau)) = \log\Big\{ \int_{\tau \in (0,1)} \exp[\mathcal{K}] \mathrm{d}\tau \Big\} = \log\{\exp[\mathcal{K}]\} \cdot \int_{\tau \in (0,1)} \mathrm{d}\tau = \mathcal{K} \sim N(0,1)$$

The result is completely proved. ■

## 2.3 Panel reverse test: I(1) → I(0)

Consider the gaussian process (1)-(3). In this case, the null hypothesis is refereed to stationary process and the alternative to a shift from I(1) to I(0). The following statistics are proposed:

$$\mathcal{K}^{\star}_{T,N}(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \Big[ \frac{[T\tau]^{-2}}{(T-[T\tau])^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} - \mu \Big], \tag{16}$$

where

$$\mu = \mathbf{E}\Big[ \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} \Big]$$

and

$$\sigma = \sqrt{\mathbf{V}\Big[ \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} \Big]}.$$

The asymptotic distribution of the statistics defined in (16) is the object of the following result.

**Theorem 4** *Suppose that Assumption 1 is true under the null hypothesis* $\mathbf{H_0}$*. Then it results*

$$\lim_{N\to+\infty} \lim_{T\to+\infty} \mathcal{K}^{\star}_{T,N}(\tau) = \mathcal{K}^{\star}(\tau) \sim N(0,1). \tag{17}$$

**Proof.** By Theorem 3.1 in Busetti and Taylor (2004), it results

$$\lim_{T\to+\infty} \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} = \frac{\tau^{-2}\int_0^{\tau}[V_i^{***}(r)]^2 \mathrm{d}r}{(1-\tau)^{-2}\int_{\tau}^{1}[V_i^{**}(r)]^2\mathrm{d}r},$$

where

$$V_i^{**}(r) = V_i(r) - V_i(\tau) - (r-\tau)(1-\tau)^{-1}(V_i(1) - V_i(\tau))$$
$$V_i^{***}(r) = V_i(r) - r\tau^{-1}V_i(\tau)$$

and

$$V_i(r) = W_0(r) + c\left\{ \int_0^{min\{r,\tau\}} W_c(s)\mathrm{d}s + 1(r>\tau)[(r-\tau)W_c(\tau)] \right\},$$

where $W$ is a standard Wiener process. Hypotheses stated in Assumption 1 imply

$$\lim_{T\to+\infty} \mathbf{E}\Big[ \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} \Big] = \bar{\mu},$$

$$\lim_{T\to+\infty} \mathbf{V}\Big[ \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} \Big] = \bar{\sigma}^2,$$

with

$$\bar{\mu} = \mathbf{E}\Big[ \frac{\tau^{-2}\int_0^{\tau}[V_i^{***}(r)]^2\mathrm{d}r}{(1-\tau)^{-2}\int_{\tau}^{1}[V_i^{**}(r)]^2\mathrm{d}r} \Big] \tag{18}$$

and

$$\bar{\sigma}^2 = \mathbf{V}\Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2 \mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2 \mathrm{d}r}\Big] \tag{19}$$

for each $i = 1, \dots, N$.
Hence

$$\lim_{N\to+\infty}\lim_{T\to+\infty} \mathcal{K}^\star_{T,N}(\tau) = \lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2 \mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2 \mathrm{d}r} - \bar{\mu}\Big],$$

for $i = 1, \dots, N$.
The Central Limit Theorem guarantees that

$$\lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^N \Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2 \mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2 \mathrm{d}r} - \bar{\mu}\Big] \sim N(0,1),$$

and the Theorem is completely proved. ■
In the next result, the asymptotic distributions of the transformations $H_1$, $H_2$ and $H_3$ of the test $\mathcal{K}^\star$ are given.

**Theorem 5** *The following propositions hold.*

*(i) It results*

$$\lim_{T\to+\infty}\lim_{N\to+\infty} H_j(\mathcal{K}^\star_{T,N}(\hat{\tau})) = H_j(\mathcal{K}^\star(\tau)), \qquad j = 1,2,3.$$

*(ii) For each $j = 1,2,3$, we have $H_j(\mathcal{K}^\star(\tau)) \sim N(0,1)$.*

**Proof.** Analogous to the proof of Theorem 3. ■

### 2.4 Panel tests with unknown direction

We now discuss the case of unknown direction of changes in persistence. Three panel tests are developed and their asymptotic distributions are derived. The tests are:

$$\mathcal{M}^{j,*}_{T,N} = \frac{\sqrt{N}}{\sigma^*_j} \cdot \frac{1}{N} \cdot \sum_{i=1}^N [\max\{H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}^*_{T,i})\} - \mu^*_j], \qquad j = 1,2,3; \tag{20}$$

where

$$\tilde{\mathcal{K}}_{T,i} = \frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \frac{\sum_{t=[T\tau]+1}^T S^{(1)}_{i,t}(\tau)^2}{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2},$$

$$\tilde{\mathcal{K}}^\star_{T,i} = (\tilde{\mathcal{K}}_{T,i})^{-1},$$

$$\mu^*_j = \mathbf{E}\Big[H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}^*_{T,i})\}\Big] \qquad j = 1,2,3; \quad i = 1, \dots, N,$$

$$\sigma^*_j = \sqrt{\mathbf{V}\Big[H_j(\tilde{\mathcal{K}}_{T,i}), H_j(\tilde{\mathcal{K}}^*_{T,i})\}\Big]} \qquad j = 1,2,3; \quad i = 1, \dots, N.$$

The asymptotic distributions of these tests are now derived.

**Theorem 6** *It results*

$$\lim_{N\to+\infty}\lim_{T\to+\infty} \mathcal{M}^{j,*}_{T,N} \sim N(0,1).$$

**Proof.** It is a direct consequence of the Central Limit Theorem. ■

## 2.5 Modified panel tests

In this section we propose panel tests that are based on the modified version of the statistics developed in subsections 2.2-2.4. These tests have the same critical value in the limit as the corresponding unmodified tests under the null hypothesis $\mathbf{H_0}$, and the same limiting critical value is also appropriate under the the alternative hypothesis $\mathbf{H_1}$. The modification proposed has no asymptotic effect under the null $\mathbf{H_0}$, so that the limiting distribution of the modified tests is the same of the corresponding unmodified tests. Under the alternative hypothesis, the asymptotic distribution of the tests is affected by this modification, but the last is chosen such that the limiting critical value is precisely the same as under the null. The panel tests developed are:

$$MH_j^d(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{1,N,T}) \cdot H_j(\mathcal{K}_{T,N}(\tau)), \qquad j = 1,2,3; \tag{21}$$

$$MH_j^d(\mathcal{K}^\star_{T,N}(\tau)) := \exp(-bJ_{1,N,T}) \cdot H_j(\mathcal{K}^\star_{T,N}(\tau)), \qquad j = 1,2,3; \tag{22}$$

$$M\mathcal{M}^{j*}_{T,N} := \exp(-bJ_{1,N,T}) \cdot \mathcal{M}^{j*}_{T,N} \qquad j = 1,2,3; \tag{23}$$

where $b$ is a finite constant and $J_{1,N,T}$ is the arithmetic mean on $N$ of the truncated sequences of $T^{-1}$ times the Wald statistic $J^{(i)}_{1,T}$ for testing the joint hypothesis $\varsigma_{i,k+1} = \cdots = \varsigma_{i,9} = 0$ in panel regression

$$y_{i,t} = \varepsilon_{i,t} + \sum_{j=k+1}^{9} \varsigma_{i,j} t^j + \text{error}, \qquad t = [\tau T]+1, \ldots, T; \;\; i = 1, \ldots, N. \tag{24}$$

Under the null hypothesis, Harvey *et al.* (2006) show that

$$\lim_{T\to+\infty} J^{(i)}_{1,T} = 1, \qquad \forall\, i = 1, \ldots, N.$$

Therefore, since we assumed independence and identical distribution with respect to the cross-sectional dimension $i$, we have

$$\lim_{T,N\to+\infty} J_{1,N,T} = \lim_{T,N\to+\infty} \frac{1}{N} \sum_{i=1}^{N} J^{(i)}_{1,T} = 1.$$

Under the alternative hypothesis, following Harvey *et al.* (2006), we modify the tests (21), (22) and (23) by introducing

$$J_{N,min} := \min_{\tau\in(0,1)} J_{1,N,[\tau T]}.$$

We define

$$MH_{j,min}(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{N,min}) \cdot H_j(\mathcal{K}_{T,N}(\tau)), \qquad j = 1,2,3; \tag{25}$$

$$MH_{j,min}(\mathcal{K}^\star_{T,N}(\tau)) := \exp(-bJ_{N,min}) \cdot H_j(\mathcal{K}^\star_{T,N}(\tau)), \qquad j = 1,2,3; \tag{26}$$

$$M\mathcal{M}^{j*}_{min,T,N} := \exp(-bJ_{N,min}) \cdot \mathcal{M}^{j*}_{T,N} \qquad j = 1,2,3; \tag{27}$$

and

$$J_{min} := \lim_{N\to+\infty} J_{N,min}.$$

By rewriting the asymptotic analysis under the alternative hypothesis of Harvey *et al.* (2006), by using the fact that the asymptotic distributions of the tests $H_j(\mathcal{K}_{T,N}(\tau))$, $H_j(\mathcal{K}^{\star}_{T,N}(\tau))$, and $\mathcal{M}^{j*}_{T,N}$, $j = 1, 2, 3$ are standard gaussian (see Theorem 3, Theorem 5 and Theorem 6), for each $j = 1, 2, 3$, we have

$$\lim_{N\to+\infty} T^{-2}(MH_{j,min}(\mathcal{K}_{T,N}(\tau)) - MH^d_j(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} (MH_{j,min}(\mathcal{K}_{T,N}(\tau)) - MH^d_j(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} \{1 - \exp[-bJ_{N,min}]\} MH^d_j(\mathcal{K}_{T,N}(\tau)) =$$

$$= T^{-2}\{1 - \exp[-bJ_{min}]\} \lim_{N\to+\infty} MH^d_j(\mathcal{K}_{T,N}(\tau)) = o_p(1)O_p(1) = o_p(1).$$

Analogously, it results

$$\lim_{N\to+\infty} T^{-2}(MH_{j,min}(\mathcal{K}^{\star}_{T,N}(\tau)) - MH^d_j(\mathcal{K}^{\star}_{T,N}(\tau))) = o_p(1)$$

and

$$\lim_{N\to+\infty} T^{-2}(M\mathcal{M}^{j*}_{min,T,N} - M\mathcal{M}^{j*}_{T,N}) = o_p(1).$$

The tests is then consistent under the alternative hypothesis.

## 2.6 Estimation of the break

In this subsection we present a procedure to estimate the unknown change point. Consider the following estimator:

$$\Lambda_{N,T}(\tau) = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{t=[T\tau]+1}^{T} \tilde{\mu}^2_{i,t}/(T - [T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}^2_{i,t}/[\tau T]}. \tag{28}$$

In order to explore the asymptotic behavior of the estimated unknown change point, the following assumption is required.

**Assumption 7** *Let $\tilde{\mu}_{i,s+1}, \tilde{\mu}_{i,s+2}, \ldots, \tilde{\mu}_{i,s+m}$, for $s \in 0, \ldots, T-1$ and $m \leq T - s$ be a sequence of stationary variables. Assume that $m^{-1}\sum_{t=s+1}^{s+m} \tilde{\mu}^2_{i,t} \to E[\mu^2_i]$ for $E[\mu^2_i] < \infty$, $\forall i = 1, \ldots, N$.*

Now, let $\hat{\tau}$ be such that:

$$\hat{\tau} = \Big\{\mathrm{argmax}_{\tau\in(0,1)} \Lambda_{\mathrm{N,T}}(\tau)\Big\}. \tag{29}$$

The following theorem shows asymptotic properties of $\hat{\tau}$:

**Theorem 8** *Suppose that Assumption 7 holds. Under the alternative hypothesis, it results*

$$(\hat{\tau} - \tau) = o_p(1), \tag{30}$$

$$T(\hat{\tau} - \tau) = O_p(1), \tag{31}$$

**Proof.** Since

$$M_T^{(i)}(\tau) := \frac{\sum_{t=[T\tau]+1}^{T} \tilde{\mu}_{i,t}^2/(T-[T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}_{i,t}^2/[\tau T]} \geq 0, \qquad \forall\, i = 1, \ldots, N,$$

then

$$\hat{\tau} = \text{argmax}_{\tau\in(0,1)} M_T^{(i)}(\tau) \Rightarrow$$

$$\Rightarrow \hat{\tau} = \text{argmax}_{\tau\in(0,1)} \left\{ \frac{1}{N} \cdot \sum_{i=1}^{N} M_T^{(i)}(\tau) \right\} = \text{argmax}_{\tau\in(0,1)} \Lambda_{N,T}(\tau).$$

Therefore, Theorem 3.5 in Kim (2000) guarantees the thesis. ■

# 3 Persistence test with cross-section correlation

Previous derivations are valid under the assumption that the units are cross-section independent. However, this requirement is rarely likely to be satisfied in empirical economic applications where the countries or regions depend each other. In order to generalize the framework of the paper we have extended our approach to account for the presence of common factors as in Stock and Watson (2002), Bai (2003) and Bai and Ng (2004).
Let as before $y_{i,t}$ be the observation on the $i$-th cross section unit at time t for $i = 1, \ldots, N$, $t = 1, \ldots, T$ and suppose that it is generated according to the following linear heterogeneous panel data model:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + \varepsilon_{i,t} \tag{32}$$

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [\tau T])\eta_{i,t}, \; if \; \tau \in (0,1) \tag{33}$$

$$\varepsilon_{i,t} = F_t \lambda_i + u_{i,t} \tag{34}$$

where $F_t$ denotes a stationary $(1 \times m)$-vector of unobserved common factors, $\lambda_i$ indicates the vector of loadings and $u_{i,t}$ is a stationary process. For the present, the deterministic component $d_{i,t}$ is taken to be the unity vector. The following assumptions are required.

**Assumption 9** *(i) for non-random* $\lambda_i, \|\lambda_i\| \leq M$*; for random* $\lambda_i$*,* $E\|\lambda_i\|^4 \leq M$*,*
*(ii)* $\frac{1}{N}\sum_{i=0}^{N} \lambda_i \lambda_i' \Rightarrow \sum_{\Pi}$*, a* $(m \times m)$ *positive matrix.*

**Assumption 10** *The error* $u_{i,t}$*, the factor* $F_t$ *and the loadings* $\lambda_i$ *are mutually independent.*

Assumption 9 ensures that the factor loadings are identifiable. The estimation of the common factors are done as in Stock and Watson (2002), i.e. using principal components. Specifically, the principal component of $F = (F_1, F_2, ..., F_T)$, denoted as $\tilde{F}$, is $\sqrt{T}$ times the first $r$ eigenvectors corresponding to the first $r$ largest eigenvalues of the $(T \times T)$ matrix of demeaned and stardardized $\tilde{y}_i \tilde{y}_i{}'$. Under the normalization $\tilde{F}\tilde{F}'/T = I_r$, the estimated loading matrix is $\tilde{\prod} = \tilde{F}'\tilde{y}_i/T$. Therefore, the estimated residuals are defined as

$$\tilde{z}_{i,t} = \tilde{y}_{i,t} - \tilde{F}_t \tilde{\lambda}_i \tag{35}$$

From the data generating process (32)-(34) and (35), it can be easily seen that for each cross section $i$, the process $\tilde{z}_{i,t}$ is stationary up to and including time $[\tau T]$ but is I(1) after the break, if and only if $\sigma^2_{\eta i} > 0$.
Thus our strategy is to apply the panel test statistics presented in section 2 to the de-factored data $\tilde{z}_{i,t}$.

# 4 Monte Carlo simulation results

In this section we use Monte Carlo experiments to examine finite sample properties of the panel persistence tests. We consider two sets of Monte Carlo experiments. The first set focuses on the model (1)-(2), i.e where we assume cross-section independence, while the second set of experiments is based on the model (32)-(35) where we allow for the presence of dependence across the different units in the panel. We start the analysis considering the empirical rejection frequencies of the tests when the data are generated according to the I(0)-I(1) switch data generating process embraced in (1)-(2) under the hypothesis of cross-sectional independency. As in Busetti and Taylor (2004) we investigate the impact of varying the signal-noise-ratio among $\sigma_{\eta i} = 0, 0.5, 0.10, 0.25, 0.50$ and $\sigma_{\varepsilon i} \sim U[0.5, 1.5]$ and the breakpoint among $\tau = 0.3, 0.5, 0.7$. The simulation results were performed 1000 Monte Carlo replications and the RNDN function of Gauss 6.0. As is often used in the literature, for all the tests we fix $\bigwedge = [0.2, 0.8]$ and $T = 50, 100$ and $N = 1, 10, 30, 50$. In Table 1 we present the moments of Kim's (2000, 2002) and Busetti and Taylor's (2004) tests which have been used to standardize the panel tests. Their values have been computed using 50000 replications.

Table 1 about here

The size results for the benchmark model (1)-(2) are reported in table 2.

Table 2 about here

All the panel test statistics seem to have good size for both small and large $T, N$. Looking at power of the tests, see table 3, table 4 and table 5 many interesting things emerge.

Table 3 about here
Table 4 about here
Table 5 about here

First of all, panel tests have better properties than single time series tests. Comparing the power of panel tests derived along the line of Kim (2000), Busetti and Taylor (2004) and Harvey *et al.* (2006) we don't find significant differences. As expected, the power of tests grows, with the exception of reversed panel tests, as the signal to noise ratio rise and the smaller is $\tau$. This occurs because the higher is $\sigma_\eta$ the stronger is the random walk component. We have that the smaller is $\tau$, the greater is the proportion of the sample containing a random walk component. Finally, the previous finding are reversed for the panel reversed tests. This depends on because we are testing a change from I(0) to I(1). We do not report for brevity the results for the cases of changes from I(1) to I(0), they are available upon request. Here as expected, the power of tests grows largely for the reversed tests and it is striking that the results mimic those in Tables 3-5. Thus $H_1(\mathcal{K}^*)$, $H_2(\mathcal{K}^*)$ and $H_3(\mathcal{K}^*)$ show better properties than $H_1(\mathcal{K})$, $H_2(\mathcal{K})$ $H_3(\mathcal{K})$ and $\mathcal{M}^{1*}$, $\mathcal{M}^{2*}$ and $\mathcal{M}^{3*}$.

We now present the empirical size of tests when cross-section dependence is included in the model as in equations (32)-(34). We consider two levels of cross section dependence where we generate $\lambda_i \sim iidU[0, 0.20]$ as an example of "low cross section dependence", and $\lambda_i \sim iidU[-1, 3]$ to represent the case of "high cross section dependence". The results are reported respectively in Table 6 and 7. As to be expected the extent of over-rejection of the tests very much depend on the degree of cross section dependence. Both for low as well as for strong cross section dependence the panel tests are distorted with over-rejection which grows as the degree of cross section dependence rise. Thus panel tests that do not allow for cross section dependence can be seriously biased if the degree of cross section dependence is large.

Table 6 about here
Table 7 about here

As previously reported to take into account of cross section dependence we use the method propose in Stock and Watson (2002). The method basically consists in filtering out the individual-specific cross-sections $y_{it}$ by the factor component computed using the principal component method. The number of factors are computed using a methodology proposed in Bai and Ng (2002). Specifically throughout the Monte Carlo simulations analysis the number of factors are computed using the $IC(3)$ criterion proposed in Bai and Ng (2002) with a maximum number of five factors.

In Tables 8-11 we present respectively the size and power of defactored panel tests using the Stock and Watson (2002) methodology. Looking at the results we note first that the tests have now generally good size, although some sign of oversize are noted especially for $\mathrm{H_j}(\mathcal{K}^*)$ tests. As expected, the power of tests grows for larger values of $T$ and $N$.

# 5 Empirical applications

We apply the panel tests described in this paper to a panel of 15 European quarterly inflation rate series observed for the period 1970.1-2006.2 [1]. The series are calculate as first difference of the logarithm of the (seasonally adjusted) consumer price index. The data are taken from OECD Main Economic Indicators. In table 12 panel tests results are reported. Looking at the results we note first that for first set of test statistics, i.e. test statistics which are computed not taking into account possible cross-sectional dependence, reverse tests strongly suggest a change of persistence from I(1) to I(0) process. Change in persistence are also evidenced by $\mathcal{M}^j$ tests as well as their modified version. Mixed response

[1]The countries included in the panel are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, United Kingdom.

are obtained from $\mathrm{H}_1(\mathcal{K})$ and $\mathrm{H}_2(\mathcal{K})$ test statistics. While the former tests reject the null hypothesis for a change from I(0) to I(1) the latter tests do not reject the null hypothesis for a process which I(0) or I(1) throughout. In order to take into account for possible cross-dependence across the countries, we first compute the number of factors. The $IC(3)$ criterion suggests three factors. Thus we use the estimated factors and factor loadings to compute $\tilde{\hat{z}}_{i,t}$ as in equation (35). Looking at the results of panel test statistics we note that the previous results are now partially reversed. Here both $\mathrm{H}_1(\mathcal{K})$ and $\mathrm{H}_2(\mathcal{K})$ test statistics do not reject the null hypothesis allowing for a process which is I(0) or I(1) throughout the sample of analysis. Reversed test statistics indicate a change from I(1) to I(0) and $\mathcal{M}^j$ strongly reject the null hypothesis of constant persistence (either constant I(0) or constant I(1)). Given the previous results we conclude that the inflation process is characterized by a change in persistence from I(1) to I(0). Using the cross-sectional dependence adjusted series, the change of persistence, computed using (29), was around the 1984.4.

# 6 Conclusions

In this paper we present new panel persistence change tests which are based on modified time series version of the ratio-based statistics by Busetti and Taylor [2004. Test of stationarity against a change in persistence. Journal of Econometrics 123, 33-66]. These statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1) or viceversa. Alternative of unknown direction is also considered. Asymptotic distributions of the new panel tests under the hypothesis of cross-section independence are derived and Monte Carlo analysis suggest that these tests perform very well. Cross-section dependence is also considered.

We show first that when testing for a change in persistence from I(0) (I(1)) to I(1) (I(0)) panel tests have better properties than the single time series tests. Secondly, we report the importance of taking into account for possible cross-sectional dependence when computing the panel test statistics, especially for highly dependent panels. Finally, we apply the panel tests to a panel of EU 15 inflation rates observed during the period 1970.1 - 2006.2. The outcomes were consistent with a change of persistence from I(1) to I(0) around the 1984.4.

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Table 1: Simulated moments for individual Kim(2000, 2002) and Busetti and Taylor tests (2004)

| T | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|
| Mean - drift case | | | | | | | | | |
| 50 | 1.817 | 1.608 | 6.173 | 1.836 | 1.624 | 6.204 | 2.783 | 2.632 | 9.214 |
| 100 | 1.803 | 1.566 | 6.386 | 1.803 | 1.564 | 6.391 | 2.749 | 2.545 | 9.436 |
| 500 | 1.800 | 1.537 | 6.788 | 1.779 | 1.506 | 6.660 | 2.741 | 2.478 | 9.868 |
| Mean - linear trend case | | | | | | | | | |
| 50 | 1.415 | 0.906 | 3.785 | 1.412 | 0.908 | 3.796 | 1.992 | 1.334 | 5.297 |
| 100 | 1.377 | 0.844 | 3.738 | 1.374 | 0.843 | 3.831 | 1.924 | 1.222 | 5.145 |
| 500 | 1.362 | 0.815 | 3.718 | 1.353 | 0.822 | 3.839 | 1.886 | 1.177 | 5.115 |
| Std. deviation - drift case | | | | | | | | | |
| 50 | 1.580 | 2.282 | 5.856 | 1.586 | 2.370 | 6.003 | 1.737 | 2.903 | 6.874 |
| 100 | 1.548 | 2.178 | 5.841 | 1.526 | 2.106 | 5.728 | 1.669 | 2.644 | 6.590 |
| 500 | 1.528 | 2.170 | 6.041 | 1.503 | 1.980 | 5.701 | 1.667 | 2.562 | 6.318 |
| Std. deviation - linear trend case | | | | | | | | | |
| 50 | 0.869 | 0.863 | 2.759 | 0.866 | 0.861 | 2.777 | 0.869 | 1.030 | 3.068 |
| 100 | 0.814 | 0.687 | 2.451 | 0.798 | 0.685 | 2.447 | 0.794 | 0.781 | 2.618 |
| 500 | 0.764 | 0.607 | 2.428 | 0.734 | 0.651 | 2.401 | 0.709 | 0.701 | 2.604 |

Table 2: Size of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 1 | 0.061 | 0.052 | 0.052 | 0.059 | 0.054 | 0.048 | 0.056 | 0.049 | 0.050 |
| | 10 | 0.054 | 0.056 | 0.061 | 0.057 | 0.074 | 0.054 | 0.057 | 0.071 | 0.054 |
| | 20 | 0.071 | 0.063 | 0.068 | 0.045 | 0.046 | 0.051 | 0.058 | 0.055 | 0.055 |
| | 50 | 0.059 | 0.061 | 0.063 | 0.053 | 0.058 | 0.056 | 0.064 | 0.068 | 0.061 |
| 100 | 1 | 0.068 | 0.072 | 0.061 | 0.064 | 0.064 | 0.054 | 0.069 | 0.071 | 0.060 |
| | 10 | 0.071 | 0.073 | 0.070 | 0.065 | 0.058 | 0.067 | 0.052 | 0.062 | 0.052 |
| | 20 | 0.054 | 0.055 | 0.052 | 0.072 | 0.073 | 0.085 | 0.068 | 0.056 | 0.074 |
| | 50 | 0.057 | 0.058 | 0.055 | 0.051 | 0.052 | 0.055 | 0.062 | 0.065 | 0.066 |

| T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 1 | 0.078 | 0.072 | 0.065 | 0.067 | 0.064 | 0.060 | 0.075 | 0.066 | 0.070 |
| | 10 | 0.059 | 0.063 | 0.059 | 0.076 | 0.062 | 0.062 | 0.057 | 0.072 | 0.060 |
| | 20 | 0.075 | 0.065 | 0.069 | 0.049 | 0.047 | 0.054 | 0.061 | 0.055 | 0.063 |
| | 50 | 0.060 | 0.061 | 0.064 | 0.054 | 0.061 | 0.057 | 0.068 | 0.069 | 0.064 |
| 100 | 1 | 0.070 | 0.076 | 0.064 | 0.072 | 0.069 | 0.062 | 0.080 | 0.081 | 0.072 |
| | 10 | 0.073 | 0.076 | 0.074 | 0.065 | 0.059 | 0.069 | 0.054 | 0.066 | 0.052 |
| | 20 | 0.055 | 0.057 | 0.052 | 0.073 | 0.073 | 0.088 | 0.072 | 0.057 | 0.074 |
| | 50 | 0.057 | 0.058 | 0.055 | 0.051 | 0.052 | 0.055 | 0.062 | 0.065 | 0.066 |

| T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $H_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_{\mathrm{m}}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 1 | 0.069 | 0.065 | 0.057 | 0.064 | 0.060 | 0.057 | 0.069 | 0.062 | 0.067 |
| | 10 | 0.055 | 0.057 | 0.063 | 0.059 | 0.075 | 0.059 | 0.061 | 0.072 | 0.060 |
| | 20 | 0.072 | 0.065 | 0.069 | 0.049 | 0.047 | 0.053 | 0.060 | 0.055 | 0.061 |
| | 50 | 0.060 | 0.061 | 0.063 | 0.054 | 0.059 | 0.056 | 0.066 | 0.069 | 0.064 |
| 100 | 1 | 0.070 | 0.073 | 0.061 | 0.066 | 0.065 | 0.058 | 0.072 | 0.076 | 0.063 |
| | 10 | 0.071 | 0.075 | 0.071 | 0.065 | 0.059 | 0.067 | 0.054 | 0.064 | 0.052 |
| | 20 | 0.055 | 0.055 | 0.052 | 0.072 | 0.073 | 0.086 | 0.068 | 0.056 | 0.074 |
| | 50 | 0.057 | 0.058 | 0.055 | 0.051 | 0.052 | 0.055 | 0.062 | 0.065 | 0.066 |

**Notes :** Empirical sizes corresponding to a 5% nominal size.

Table 3: Power of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 1 | 0.095 | 0.100 | 0.090 | 0.059 | 0.063 | 0.056 | 0.087 | 0.086 | 0.085 |
| | | | 10 | 0.126 | 0.139 | 0.135 | 0.078 | 0.068 | 0.083 | 0.112 | 0.125 | 0.116 |
| | | | 20 | 0.156 | 0.181 | 0.167 | 0.074 | 0.052 | 0.078 | 0.139 | 0.154 | 0.141 |
| | | | 50 | 0.260 | 0.256 | 0.268 | 0.084 | 0.065 | 0.092 | 0.205 | 0.231 | 0.219 |
| | | 100 | 1 | 0.200 | 0.198 | 0.195 | 0.115 | 0.090 | 0.101 | 0.196 | 0.198 | 0.192 |
| | | | 10 | 0.419 | 0.443 | 0.432 | 0.171 | 0.113 | 0.192 | 0.405 | 0.392 | 0.432 |
| | | | 20 | 0.534 | 0.558 | 0.553 | 0.208 | 0.124 | 0.235 | 0.511 | 0.528 | 0.554 |
| | | | 50 | 0.828 | 0.850 | 0.860 | 0.288 | 0.126 | 0.347 | 0.818 | 0.837 | 0.864 |
| 0.5 | | 50 | 1 | 0.083 | 0.108 | 0.075 | 0.053 | 0.053 | 0.046 | 0.076 | 0.081 | 0.073 |
| | | | 10 | 0.126 | 0.152 | 0.133 | 0.047 | 0.047 | 0.045 | 0.087 | 0.110 | 0.095 |
| | | | 20 | 0.155 | 0.180 | 0.155 | 0.049 | 0.031 | 0.052 | 0.100 | 0.124 | 0.103 |
| | | | 50 | 0.229 | 0.288 | 0.244 | 0.039 | 0.025 | 0.041 | 0.144 | 0.167 | 0.148 |
| | | 100 | 1 | 0.191 | 0.204 | 0.185 | 0.055 | 0.049 | 0.053 | 0.166 | 0.171 | 0.167 |
| | | | 10 | 0.379 | 0.454 | 0.396 | 0.058 | 0.037 | 0.065 | 0.278 | 0.308 | 0.304 |
| | | | 20 | 0.486 | 0.593 | 0.507 | 0.055 | 0.026 | 0.064 | 0.348 | 0.399 | 0.377 |
| | | | 50 | 0.804 | 0.884 | 0.826 | 0.022 | 0.002 | 0.038 | 0.568 | 0.678 | 0.617 |
| 0.7 | | 50 | 1 | 0.077 | 0.081 | 0.068 | 0.057 | 0.045 | 0.048 | 0.070 | 0.070 | 0.062 |
| | | | 10 | 0.085 | 0.115 | 0.090 | 0.050 | 0.052 | 0.048 | 0.076 | 0.096 | 0.079 |
| | | | 20 | 0.106 | 0.126 | 0.107 | 0.045 | 0.034 | 0.051 | 0.084 | 0.092 | 0.085 |
| | | | 50 | 0.127 | 0.177 | 0.140 | 0.041 | 0.032 | 0.043 | 0.085 | 0.111 | 0.088 |
| | | 100 | 1 | 0.131 | 0.153 | 0.126 | 0.055 | 0.040 | 0.051 | 0.113 | 0.131 | 0.110 |
| | | | 10 | 0.217 | 0.316 | 0.233 | 0.047 | 0.037 | 0.048 | 0.145 | 0.200 | 0.161 |
| | | | 20 | 0.265 | 0.379 | 0.297 | 0.036 | 0.028 | 0.043 | 0.148 | 0.222 | 0.172 |
| | | | 50 | 0.455 | 0.660 | 0.503 | 0.004 | 0.002 | 0.009 | 0.216 | 0.375 | 0.245 |
| 0.3 | 0.10 | 50 | 1 | 0.166 | 0.172 | 0.151 | 0.109 | 0.085 | 0.102 | 0.167 | 0.170 | 0.163 |
| | | | 10 | 0.440 | 0.448 | 0.444 | 0.180 | 0.123 | 0.194 | 0.412 | 0.414 | 0.429 |
| | | | 20 | 0.570 | 0.587 | 0.580 | 0.208 | 0.109 | 0.236 | 0.545 | 0.529 | 0.567 |
| | | | 50 | 0.863 | 0.866 | 0.867 | 0.329 | 0.136 | 0.354 | 0.843 | 0.824 | 0.856 |
| | | 100 | 1 | 0.451 | 0.429 | 0.447 | 0.262 | 0.177 | 0.255 | 0.477 | 0.449 | 0.479 |
| | | | 10 | 0.948 | 0.928 | 0.949 | 0.572 | 0.323 | 0.597 | 0.949 | 0.927 | 0.953 |
| | | | 20 | 0.995 | 0.995 | 0.996 | 0.747 | 0.405 | 0.774 | 0.993 | 0.984 | 0.993 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.950 | 0.639 | 0.965 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.148 | 0.180 | 0.141 | 0.056 | 0.046 | 0.050 | 0.125 | 0.143 | 0.124 |
| | | | 10 | 0.407 | 0.476 | 0.424 | 0.047 | 0.029 | 0.057 | 0.275 | 0.337 | 0.282 |
| | | | 20 | 0.512 | 0.617 | 0.534 | 0.049 | 0.019 | 0.057 | 0.341 | 0.411 | 0.357 |
| | | | 50 | 0.823 | 0.892 | 0.835 | 0.026 | 0.008 | 0.036 | 0.597 | 0.684 | 0.642 |
| | | 100 | 1 | 0.401 | 0.433 | 0.400 | 0.099 | 0.044 | 0.089 | 0.353 | 0.387 | 0.362 |
| | | | 10 | 0.917 | 0.952 | 0.926 | 0.120 | 0.013 | 0.148 | 0.853 | 0.882 | 0.872 |
| | | | 20 | 0.981 | 0.997 | 0.988 | 0.128 | 0.008 | 0.160 | 0.952 | 0.974 | 0.961 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.106 | 0.002 | 0.144 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.103 | 0.133 | 0.099 | 0.044 | 0.038 | 0.039 | 0.085 | 0.101 | 0.084 |
| | | | 10 | 0.221 | 0.320 | 0.227 | 0.034 | 0.030 | 0.033 | 0.129 | 0.193 | 0.137 |
| | | | 20 | 0.249 | 0.400 | 0.277 | 0.022 | 0.013 | 0.031 | 0.150 | 0.226 | 0.159 |
| | | | 50 | 0.476 | 0.649 | 0.492 | 0.012 | 0.011 | 0.015 | 0.231 | 0.383 | 0.247 |
| | | 100 | 1 | 0.285 | 0.341 | 0.271 | 0.036 | 0.026 | 0.029 | 0.220 | 0.263 | 0.225 |
| | | | 10 | 0.698 | 0.826 | 0.719 | 0.015 | 0.008 | 0.023 | 0.499 | 0.662 | 0.545 |
| | | | 20 | 0.839 | 0.935 | 0.865 | 0.007 | 0.002 | 0.015 | 0.637 | 0.826 | 0.685 |
| | | | 50 | 0.996 | 1.000 | 0.998 | 0.000 | 0.001 | 0.003 | 0.895 | 0.986 | 0.940 |

Table 3: continued

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 1 | 0.507 | 0.512 | 0.505 | 0.276 | 0.191 | 0.269 | 0.538 | 0.513 | 0.535 |
| | | | 10 | 0.988 | 0.982 | 0.987 | 0.724 | 0.420 | 0.728 | 0.987 | 0.983 | 0.986 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.850 | 0.570 | 0.866 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.994 | 0.995 | 0.994 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.821 | 0.780 | 0.814 | 0.430 | 0.300 | 0.419 | 0.826 | 0.792 | 0.825 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.931 | 0.751 | 0.938 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.995 | 0.901 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.487 | 0.541 | 0.476 | 0.111 | 0.047 | 0.101 | 0.448 | 0.476 | 0.452 |
| | | | 10 | 0.975 | 0.988 | 0.974 | 0.170 | 0.025 | 0.187 | 0.936 | 0.962 | 0.939 |
| | | | 20 | 0.999 | 0.999 | 0.999 | 0.179 | 0.011 | 0.208 | 0.999 | 0.999 | 0.999 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.255 | 0.003 | 0.310 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.784 | 0.812 | 0.782 | 0.213 | 0.079 | 0.200 | 0.753 | 0.762 | 0.752 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.508 | 0.078 | 0.532 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.648 | 0.062 | 0.684 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.873 | 0.032 | 0.908 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.333 | 0.410 | 0.337 | 0.022 | 0.016 | 0.019 | 0.251 | 0.356 | 0.261 |
| | | | 10 | 0.847 | 0.932 | 0.857 | 0.014 | 0.005 | 0.014 | 0.689 | 0.840 | 0.714 |
| | | | 20 | 0.957 | 0.987 | 0.961 | 0.002 | 0.000 | 0.004 | 0.827 | 0.947 | 0.845 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.00 | 0.001 | 0.987 | 0.998 | 0...992 |
| | | 100 | 1 | 0.652 | 0.711 | 0.653 | 0.012 | 0.006 | 0.009 | 0.590 | 0.650 | 0.594 |
| | | | 10 | 0.999 | 0.999 | 0.999 | 0.001 | 0.000 | 0.003 | 0.993 | 0.996 | 0.995 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 1 | 0.818 | 0.784 | 0.818 | 0.391 | 0.283 | 0.381 | 0.818 | 0.790 | 0.819 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.926 | 0.724 | 0.936 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.987 | 0.893 | 0.991 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.963 | 0.948 | 0.963 | 0.504 | 0.362 | 0.491 | 0.957 | 0.936 | 0.956 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.967 | 0.852 | 0.970 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.796 | 0.817 | 0.796 | 0.161 | 0.070 | 0.152 | 0.748 | 0.788 | 0.748 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.450 | 0.070 | 0.475 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.593 | 0.060 | 0.632 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.846 | 0.034 | 0.880 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.951 | 0.960 | 0.951 | 0.261 | 0.107 | 0.250 | 0.932 | 0.949 | 0.933 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.700 | 0.192 | 0.721 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.876 | 0.214 | 0.899 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.983 | 0.245 | 0.988 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.720 | 0.641 | 0.645 | 0.013 | 0.004 | 0.013 | 0.568 | 0.657 | 0.570 |
| | | | 10 | 0.999 | 0.999 | 0.999 | 0.004 | 0.001 | 0.005 | 0.991 | 0.999 | 0.990 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.877 | 0.920 | 0.878 | 0.012 | 0.002 | 0.010 | 0.852 | 0.897 | 0.853 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.002 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |

Table 4: Power of Modified Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 1 | 0.115 | 0.114 | 0.105 | 0.071 | 0.070 | 0.064 | 0.110 | 0.107 | 0.099 |
| | | | 10 | 0.134 | 0.143 | 0.141 | 0.088 | 0.072 | 0.084 | 0.121 | 0.132 | 0.124 |
| | | | 20 | 0.159 | 0.182 | 0.172 | 0.078 | 0.052 | 0.079 | 0.147 | 0.160 | 0.146 |
| | | | 50 | 0.265 | 0.259 | 0.274 | 0.085 | 0.066 | 0.094 | 0.213 | 0.236 | 0.224 |
| | | 100 | 1 | 0.220 | 0.206 | 0.210 | 0.098 | 0.110 | 0.125 | 0.215 | 0.212 | 0.219 |
| | | | 10 | 0.427 | 0.447 | 0.443 | 0.177 | 0.113 | 0.196 | 0.411 | 0.403 | 0.435 |
| | | | 20 | 0.539 | 0.561 | 0.557 | 0.208 | 0.125 | 0.239 | 0.516 | 0.534 | 0.560 |
| | | | 50 | 0.830 | 0.854 | 0.861 | 0.293 | 0.127 | 0.350 | 0.824 | 0.837 | 0.864 |
| 0.5 | | 50 | 1 | 0.098 | 0.118 | 0.091 | 0.066 | 0.061 | 0.056 | 0.095 | 0.104 | 0.086 |
| | | | 10 | 0.138 | 0.156 | 0.146 | 0.049 | 0.052 | 0.046 | 0.096 | 0.122 | 0.099 |
| | | | 20 | 0.157 | 0.181 | 0.159 | 0.050 | 0.032 | 0.055 | 0.103 | 0.125 | 0.107 |
| | | | 50 | 0.234 | 0.291 | 0.247 | 0.039 | 0.026 | 0.042 | 0.145 | 0.169 | 0.153 |
| | | 100 | 1 | 0.208 | 0.210 | 0.197 | 0.063 | 0.054 | 0.057 | 0.185 | 0.185 | 0.180 |
| | | | 10 | 0.389 | 0.462 | 0.398 | 0.059 | 0.037 | 0.068 | 0.291 | 0.315 | 0.309 |
| | | | 20 | 0.490 | 0.597 | 0.515 | 0.058 | 0.026 | 0.065 | 0.355 | 0.403 | 0.378 |
| | | | 50 | 0.805 | 0.886 | 0.827 | 0.023 | 0.002 | 0.038 | 0.572 | 0.682 | 0.618 |
| 0.7 | | 50 | 1 | 0.092 | 0.082 | 0.090 | 0.065 | 0.059 | 0.059 | 0.088 | 0.083 | 0.080 |
| | | | 10 | 0.091 | 0.118 | 0.096 | 0.052 | 0.054 | 0.050 | 0.080 | 0.100 | 0.083 |
| | | | 20 | 0.107 | 0.126 | 0.109 | 0.045 | 0.037 | 0.051 | 0.087 | 0.094 | 0.090 |
| | | | 50 | 0.129 | 0.178 | 0.145 | 0.041 | 0.034 | 0.044 | 0.089 | 0.115 | 0.090 |
| | | 100 | 1 | 0.142 | 0.164 | 0.131 | 0.063 | 0.047 | 0.054 | 0.130 | 0.146 | 0.124 |
| | | | 10 | 0.220 | 0.318 | 0.236 | 0.047 | 0.037 | 0.050 | 0.149 | 0.203 | 0.164 |
| | | | 20 | 0.269 | 0.381 | 0.299 | 0.036 | 0.028 | 0.044 | 0.148 | 0.227 | 0.174 |
| | | | 50 | 0.459 | 0.660 | 0.505 | 0.004 | 0.003 | 0.010 | 0.217 | 0.380 | 0.248 |
| 0.3 | 0.10 | 50 | 1 | 0.190 | 0.199 | 0.182 | 0.132 | 0.094 | 0.120 | 0.200 | 0.196 | 0.192 |
| | | | 10 | 0.452 | 0.455 | 0.454 | 0.186 | 0.128 | 0.200 | 0.427 | 0.426 | 0.441 |
| | | | 20 | 0.575 | 0.596 | 0.587 | 0.216 | 0.114 | 0.241 | 0.557 | 0.538 | 0.574 |
| | | | 50 | 0.866 | 0.866 | 0.869 | 0.333 | 0.137 | 0.360 | 0.849 | 0.826 | 0.860 |
| | | 100 | 1 | 0.480 | 0.448 | 0.474 | 0.284 | 0.193 | 0.270 | 0.512 | 0.473 | 0.508 |
| | | | 10 | 0.950 | 0.929 | 0.949 | 0.578 | 0.330 | 0.609 | 0.951 | 0.928 | 0.955 |
| | | | 20 | 0.995 | 0.995 | 0.996 | 0.750 | 0.407 | 0.775 | 0.993 | 0.984 | 0.994 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.950 | 0.640 | 0.966 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.149 | 0.188 | 0.145 | 0.056 | 0.046 | 0.051 | 0.121 | 0.143 | 0.120 |
| | | | 10 | 0.419 | 0.484 | 0.436 | 0.048 | 0.032 | 0.058 | 0.292 | 0.346 | 0.295 |
| | | | 20 | 0.521 | 0.620 | 0.542 | 0.049 | 0.020 | 0.057 | 0.356 | 0.418 | 0.368 |
| | | | 50 | 0.828 | 0.895 | 0.840 | 0.026 | 0.009 | 0.037 | 0.608 | 0.691 | 0.649 |
| | | 100 | 1 | 0.418 | 0.448 | 0.419 | 0.111 | 0.049 | 0.101 | 0.390 | 0.411 | 0.391 |
| | | | 10 | 0.920 | 0.953 | 0.931 | 0.127 | 0.014 | 0.151 | 0.860 | 0.886 | 0.876 |
| | | | 20 | 0.982 | 0.997 | 0.988 | 0.131 | 0.008 | 0.162 | 0.954 | 0.974 | 0.962 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.107 | 0.002 | 0.144 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.129 | 0.157 | 0.115 | 0.054 | 0.050 | 0.048 | 0.108 | 0.120 | 0.103 |
| | | | 10 | 0.226 | 0.323 | 0.236 | 0.034 | 0.031 | 0.035 | 0.135 | 0.200 | 0.147 |
| | | | 20 | 0.258 | 0.404 | 0.283 | 0.023 | 0.014 | 0.032 | 0.160 | 0.232 | 0.164 |
| | | | 50 | 0.480 | 0.654 | 0.494 | 0.014 | 0.011 | 0.015 | 0.238 | 0.387 | 0.254 |
| | | 100 | 1 | 0.295 | 0.360 | 0.289 | 0.038 | 0.028 | 0.032 | 0.247 | 0.284 | 0.248 |
| | | | 10 | 0.706 | 0.830 | 0.723 | 0.015 | 0.008 | 0.023 | 0.508 | 0.669 | 0.552 |
| | | | 20 | 0.868 | 0.842 | 0.935 | 0.015 | 0.007 | 0.002 | 0.640 | 0.827 | 0.692 |
| | | | 50 | 0.996 | 1.000 | 0.998 | 0.000 | 0.001 | 0.003 | 0.898 | 0.986 | 0.940 |

Table 4: continued

| $\tau$ | $\sigma_\eta$ | T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 1 | 0.557 | 0.547 | 0.546 | 0.311 | 0.226 | 0.299 | 0.597 | 0.570 | 0.588 |
| | | | 10 | 0.989 | 0.983 | 0.988 | 0.732 | 0.437 | 0.743 | 0.989 | 0.984 | 0.987 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.857 | 0.581 | 0.869 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.994 | 0.859 | 0.995 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.844 | 0.798 | 0.835 | 0.486 | 0.347 | 0.460 | 0.846 | 0.817 | 0.842 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.935 | 0.757 | 0.943 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.995 | 0.903 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.531 | 0.571 | 0.532 | 0.135 | 0.054 | 0.117 | 0.495 | 0.526 | 0.491 |
| | | | 10 | 0.976 | 0.988 | 0.976 | 0.176 | 0.027 | 0.195 | 0.939 | 0.963 | 0.944 |
| | | | 20 | 0.999 | 0.999 | 0.999 | 0.184 | 0.011 | 0.212 | 0.997 | 0.999 | 0.997 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.258 | 0.003 | 0.257 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.806 | 0.826 | 0.806 | 0.238 | 0.090 | 0.223 | 0.769 | 0.779 | 0.771 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.517 | 0.080 | 0.539 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.653 | 0.063 | 0.698 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.875 | 0.032 | 0.909 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.368 | 0.427 | 0.369 | 0.024 | 0.021 | 0.311 | 0.028 | 0.376 | 0.316 |
| | | | 10 | 0.851 | 0.934 | 0.861 | 0.011 | 0.005 | 0.015 | 0.704 | 0.847 | 0.722 |
| | | | 20 | 0.958 | 0.987 | 0.963 | 0.004 | 0.002 | 0.000 | 0.849 | 0.832 | 0.948 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.001 | 0.987 | 0.998 | 0.992 |
| | | 100 | 1 | 0.674 | 0.723 | 0.668 | 0.016 | 0.006 | 0.012 | 0.616 | 0.668 | 0.619 |
| | | | 10 | 0.999 | 1.000 | 0.999 | 0.001 | 0.000 | 0.004 | 0.994 | 0.996 | 0.995 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 1 | 0.856 | 0.821 | 0.849 | 0.440 | 0.319 | 0.419 | 0.869 | 0.836 | 0.867 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.931 | 0.739 | 0.937 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.988 | 0.894 | 0.991 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.972 | 0.957 | 0.969 | 0.538 | 0.410 | 0.524 | 0.968 | 0.955 | 0.968 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.970 | 0.860 | 0.970 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 0.972 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.820 | 0.838 | 0.819 | 0.202 | 0.091 | 0.180 | 0.796 | 0.811 | 0.795 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.464 | 0.076 | 0.494 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.602 | 0.062 | 0.645 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.847 | 0.036 | 0.883 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.959 | 0.962 | 0.957 | 0.306 | 0.136 | 0.283 | 0.944 | 0.955 | 0.942 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.707 | 0.202 | 0.727 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.879 | 0.219 | 0.904 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.983 | 0.249 | 0.988 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.672 | 0.743 | 0.672 | 0.018 | 0.007 | 0.015 | 0.616 | 0.685 | 0.615 |
| | | | 10 | 0.999 | 0.999 | 0.999 | 0.004 | 0.001 | 0.005 | 0.992 | 0.999 | 0.992 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.000 | 0.00 | 0.001 | 1.000 | 1.000 | 1...000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.877 | 0.923 | 0.890 | 0.018 | 0.003 | 0.014 | 0.869 | 0.900 | 0.871 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.002 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |

Table 5: Power of Modified min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

| $\tau$ | $\sigma\eta$ | T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_m^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 1 | 0.106 | 0.109 | 0.097 | 0.064 | 0.068 | 0.061 | 0.098 | 0.102 | 0.092 |
| | | | 10 | 0.129 | 0.142 | 0.139 | 0.083 | 0.071 | 0.084 | 0.116 | 0.129 | 0.121 |
| | | | 20 | 0.157 | 0.182 | 0.171 | 0.077 | 0.052 | 0.078 | 0.145 | 0.160 | 0.142 |
| | | | 50 | 0.263 | 0.259 | 0.272 | 0.085 | 0.065 | 0.093 | 0.211 | 0.233 | 0.222 |
| | | 100 | 1 | 0.203 | 0.198 | 0.197 | 0.117 | 0.093 | 0.103 | 0.204 | 0.203 | 0.199 |
| | | | 10 | 0.420 | 0.445 | 0.434 | 0.172 | 0.113 | 0.193 | 0.409 | 0.397 | 0.433 |
| | | | 20 | 0.535 | 0.559 | 0.553 | 0.208 | 0.125 | 0.236 | 0.513 | 0.530 | 0.556 |
| | | | 50 | 0.829 | 0.850 | 0.861 | 0.289 | 0.126 | 0.347 | 0.818 | 0.837 | 0.864 |
| 0.5 | | 50 | 1 | 0.091 | 0.115 | 0.082 | 0.061 | 0.059 | 0.053 | 0.082 | 0.100 | 0.099 |
| | | | 10 | 0.155 | 0.143 | 0.133 | 0.048 | 0.051 | 0.046 | 0.093 | 0.121 | 0.099 |
| | | | 20 | 0.181 | 0.159 | 0.156 | 0.049 | 0.032 | 0.052 | 0.102 | 0.124 | 0.106 |
| | | | 50 | 0.233 | 0.290 | 0.246 | 0.039 | 0.026 | 0.042 | 0.144 | 0.169 | 0.151 |
| | | 100 | 1 | 0.193 | 0.207 | 0.188 | 0.058 | 0.051 | 0.054 | 0.175 | 0.177 | 0.170 |
| | | | 10 | 0.382 | 0.455 | 0.396 | 0.059 | 0.037 | 0.068 | 0.282 | 0.311 | 0.307 |
| | | | 20 | 0.488 | 0.595 | 0.511 | 0.055 | 0.026 | 0.064 | 0.349 | 0.399 | 0.377 |
| | | | 50 | 0.804 | 0.885 | 0.826 | 0.022 | 0.002 | 0.038 | 0.568 | 0.680 | 0.618 |
| 0.7 | | 50 | 1 | 0.082 | 0.082 | 0.075 | 0.064 | 0.058 | 0.055 | 0.083 | 0.092 | 0.078 |
| | | | 10 | 0.089 | 0.118 | 0.093 | 0.050 | 0.054 | 0.049 | 0.080 | 0.100 | 0.081 |
| | | | 20 | 0.106 | 0.126 | 0.108 | 0.045 | 0.037 | 0.051 | 0.086 | 0.094 | 0.088 |
| | | | 50 | 0.129 | 0.178 | 0.1402 | 0.041 | 0.032 | 0.043 | 0.088 | 0.114 | 0.090 |
| | | 100 | 1 | 0.133 | 0.155 | 0.127 | 0.057 | 0.042 | 0.052 | 0.118 | 0.139 | 0.114 |
| | | | 10 | 0.218 | 0.318 | 0.234 | 0.047 | 0.037 | 0.048 | 0.147 | 0.202 | 0.161 |
| | | | 20 | 0.266 | 0.381 | 0.299 | 0.036 | 0.028 | 0.043 | 0.148 | 0.224 | 0.173 |
| | | | 50 | 0.456 | 0.660 | 0.503 | 0.004 | 0.003 | 0.010 | 0.216 | 0.377 | 0.245 |
| 0.3 | 0.10 | 50 | 1 | 0.181 | 0.188 | 0.166 | 0.120 | 0.090 | 0.109 | 0.187 | 0.179 | 0.184 |
| | | | 10 | 0.446 | 0.452 | 0.448 | 0.181 | 0.125 | 0.199 | 0.420 | 0.424 | 0.439 |
| | | | 20 | 0.570 | 0.591 | 0.584 | 0.211 | 0.112 | 0.236 | 0.551 | 0.536 | 0.573 |
| | | | 50 | 0.865 | 0.866 | 0.868 | 0.331 | 0.137 | 0.355 | 0.847 | 0.825 | 0.859 |
| | | 100 | 1 | 0.454 | 0.430 | 0.450 | 0.265 | 0.180 | 0.256 | 0.485 | 0.454 | 0.486 |
| | | | 10 | 0.948 | 0.928 | 0.949 | 0.573 | 0.323 | 0.599 | 0.950 | 0.928 | 0.954 |
| | | | 20 | 0.995 | 0.995 | 0.996 | 0.747 | 0.405 | 0.774 | 0.993 | 0.984 | 0.993 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.950 | 0.640 | 0.965 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.165 | 0.191 | 0.154 | 0.062 | 0.048 | 0.055 | 0.145 | 0.157 | 0.141 |
| | | | 10 | 0.419 | 0.484 | 0.436 | 0.047 | 0.032 | 0.058 | 0.281 | 0.342 | 0.291 |
| | | | 20 | 0.517 | 0.618 | 0.537 | 0.049 | 0.019 | 0.057 | 0.351 | 0.414 | 0.360 |
| | | | 50 | 0.826 | 0.893 | 0.836 | 0.026 | 0.009 | 0.037 | 0.605 | 0.689 | 0.647 |
| | | 100 | 1 | 0.404 | 0.437 | 0.402 | 0.101 | 0.044 | 0.093 | 0.605 | 0.389 | 0.370 |
| | | | 10 | 0.918 | 0.952 | 0.927 | 0.122 | 0.013 | 0.148 | 0.853 | 0.883 | 0.873 |
| | | | 20 | 0.981 | 0.997 | 0.988 | 0.128 | 0.008 | 0.160 | 0.952 | 0.974 | 0.961 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.107 | 0.002 | 0.144 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.119 | 0.147 | 0.104 | 0.051 | 0.047 | 0.044 | 0.100 | 0.115 | 0.097 |
| | | | 10 | 0.225 | 0.321 | 0.233 | 0.034 | 0.031 | 0.033 | 0.135 | 0.199 | 0.145 |
| | | | 20 | 0.255 | 0.402 | 0.279 | 0.022 | 0.014 | 0.032 | 0.159 | 0.231 | 0.161 |
| | | | 50 | 0.477 | 0.651 | 0.493 | 0.014 | 0.011 | 0.015 | 0.235 | 0.387 | 0.251 |
| | | 100 | 1 | 0.287 | 0.346 | 0.278 | 0.036 | 0.026 | 0.031 | 0.230 | 0.271 | 0.236 |
| | | | 10 | 0.702 | 0.828 | 0.719 | 0.015 | 0.008 | 0.023 | 0.501 | 0.665 | 0.547 |
| | | | 20 | 0.839 | 0.935 | 0.866 | 0.007 | 0.002 | 0.015 | 0.638 | 0.826 | 0.686 |
| | | | 50 | 0.996 | 1.000 | 0.998 | 0.000 | 0.001 | 0.003 | 0.896 | 0.986 | 0.940 |

Table 5: continued

| $\tau$ | $\sigma_\eta$ | T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_m^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 1 | 0.523 | 0.520 | 0.522 | 0.288 | 0.202 | 0.277 | 0.569 | 0.535 | 0.558 |
| | | | 10 | 0.988 | 0.983 | 0.987 | 0.725 | 0.424 | 0.729 | 0.988 | 0.983 | 0.987 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.857 | 0.581 | 0.869 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.994 | 0.856 | 0.995 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.825 | 0.782 | 0.816 | 0.435 | 0.306 | 0.425 | 0.833 | 0.800 | 0.829 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.931 | 0.751 | 0.938 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.995 | 0.901 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.451 | 0.553 | 0.496 | 0.135 | 0.054 | 0.117 | 0.465 | 0.497 | 0.469 |
| | | | 10 | 0.975 | 0.988 | 0.975 | 0.176 | 0.027 | 0.195 | 0.937 | 0.962 | 0.939 |
| | | | 20 | 0.999 | 0.999 | 0.999 | 0.210 | 0.180 | 0.011 | 0.996 | 0.997 | 0.999 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.258 | 0.003 | 0.3102 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.788 | 0.816 | 0.784 | 0.217 | 0.079 | 0.200 | 0.755 | 0.765 | 0.752 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.508 | 0.078 | 0.534 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.648 | 0.062 | 0.684 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.875 | 0.032 | 0.909 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.352 | 0.420 | 0.348 | 0.025 | 0.021 | 0.023 | 0.286 | 0.369 | 0.283 |
| | | | 10 | 0.851 | 0.934 | 0.861 | 0.010 | 0.005 | 0.014 | 0.699 | 0.845 | 0.718 |
| | | | 20 | 0.957 | 0.987 | 0.961 | 0.002 | 0.000 | 0.004 | 0.832 | 0.948 | 0.847 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.001 | 0.987 | 0.998 | 0.992 |
| | | 100 | 1 | 0.657 | 0.712 | 0.654 | 0.012 | 0.006 | 0.010 | 0.598 | 0.658 | 0.602 |
| | | | 10 | 0.999 | 0.999 | 0.999 | 0.001 | 0.000 | 0.003 | 0.993 | 0.996 | 0.995 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 1 | 0.830 | 0.793 | 0.823 | 0.403 | 0.293 | 0.388 | 0.845 | 0.805 | 0.838 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.928 | 0.725 | 0.936 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.987 | 0.894 | 0.991 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.997 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.963 | 0.950 | 0.964 | 0.507 | 0.364 | 0.494 | 0.958 | 0.939 | 0.957 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.967 | 0.852 | 0.970 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 1.000 | 0.969 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 1 | 0.804 | 0.830 | 0.803 | 0.169 | 0.073 | 0.157 | 0.774 | 0.799 | 0.774 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.453 | 0.071 | 0.479 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.596 | 0.061 | 0.637 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.847 | 0.034 | 0.880 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.961 | 0.953 | 0.952 | 0.266 | 0.110 | 0.251 | 0.951 | 0.934 | 0.933 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.700 | 0.192 | 0.721 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.877 | 0.215 | 0.900 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.983 | 0.249 | 0.988 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 1 | 0.655 | 0.735 | 0.654 | 0.018 | 0.007 | 0.015 | 0.590 | 0.674 | 0.590 |
| | | | 10 | 0.999 | 0.999 | 0.999 | 0.004 | 0.001 | 0.005 | 0.991 | 0.999 | 0.991 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 1 | 0.878 | 0.921 | 0.879 | 0.013 | 0.002 | 0.010 | 0.860 | 0.897 | 0.857 |
| | | | 10 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 20 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.002 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |

Table 6: Size of Panel Tests When the Model Allows for Low Cross Sectional Dependence. Intercept case.

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.178 | 0.166 | 0.166 | 0.057 | 0.055 | 0.062 | 0.166 | 0.167 | 0.162 |
| | 20 | 0.181 | 0.183 | 0.185 | 0.081 | 0.062 | 0.076 | 0.165 | 0.156 | 0.165 |
| | 50 | 0.199 | 0.190 | 0.194 | 0.083 | 0.068 | 0.076 | 0.210 | 0.204 | 0.200 |
| 100 | 10 | 0.179 | 0.176 | 0.174 | 0.070 | 0.075 | 0.070 | 0.176 | 0.173 | 0.177 |
| | 20 | 0.179 | 0.173 | 0.181 | 0.083 | 0.091 | 0.085 | 0.185 | 0.180 | 0.188 |
| | 50 | 0.229 | 0.216 | 0.227 | 0.104 | 0.090 | 0.103 | 0.231 | 0.221 | 0.235 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.180 | 0.168 | 0.176 | 0.057 | 0.066 | 0.061 | 0.169 | 0.166 | 0.168 |
| | 20 | 0.185 | 0.186 | 0.185 | 0.082 | 0.063 | 0.076 | 0.170 | 0.157 | 0.166 |
| | 50 | 0.200 | 0.191 | 0.197 | 0.085 | 0.069 | 0.077 | 0.210 | 0.207 | 0.202 |
| 100 | 10 | 0.181 | 0.176 | 0.176 | 0.070 | 0.076 | 0.075 | 0.178 | 0.177 | 0.179 |
| | 20 | 0.179 | 0.174 | 0.184 | 0.095 | 0.083 | 0.085 | 0.187 | 0.180 | 0.189 |
| | 50 | 0.230 | 0.217 | 0.228 | 0.104 | 0.090 | 0.106 | 0.231 | 0.223 | 0.238 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.179 | 0.168 | 0.175 | 0.057 | 0.064 | 0.056 | 0.169 | 0.168 | 0.168 |
| | 20 | 0.182 | 0.185 | 0.185 | 0.082 | 0.063 | 0.076 | 0.169 | 0.157 | 0.166 |
| | 50 | 0.200 | 0.191 | 0.196 | 0.085 | 0.069 | 0.077 | 0.210 | 0.207 | 0.202 |
| 100 | 10 | 0.336 | 0.332 | 0.304 | 0.160 | 0.152 | 0.139 | 0.336 | 0.332 | 0.304 |
| | 20 | 0.179 | 0.174 | 0.183 | 0.095 | 0.083 | 0.085 | 0.185 | 0.180 | 0.188 |
| | 50 | 0.229 | 0.217 | 0.227 | 0.104 | 0.090 | 0.105 | 0.231 | 0.222 | 0.236 |

**Notes :** Empirical sizes corresponding to a 5% nominal size.

Table 7: Sizes of Panel Tests When the Model Allows for Strong Cross Sectional Dependence. Intercept case.

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.360 | 0.361 | 0.333 | 0.211 | 0.182 | 0.180 | 0.358 | 0.335 | 0.342 |
| | 20 | 0.419 | 0.409 | 0.381 | 0.230 | 0.205 | 0.200 | 0.402 | 0.381 | 0.391 |
| | 50 | 0.477 | 0.458 | 0.433 | 0.255 | 0.246 | 0.215 | 0.425 | 0.406 | 0.425 |
| 100 | 10 | 0.335 | 0.332 | 0.304 | 0.160 | 0.152 | 0.138 | 0.324 | 0.292 | 0.311 |
| | 20 | 0.365 | 0.360 | 0.334 | 0.220 | 0.210 | 0.180 | 0.355 | 0.331 | 0.320 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.367 | 0.362 | 0.337 | 0.213 | 0.183 | 0.183 | 0.361 | 0.339 | 0.344 |
| | 20 | 0.420 | 0.410 | 0.383 | 0.230 | 0.208 | 0.203 | 0.409 | 0.402 | 0.384 |
| | 50 | 0.478 | 0.458 | 0.434 | 0.257 | 0.246 | 0.215 | 0.443 | 0.406 | 0.427 |
| 100 | 10 | 0.339 | 0.335 | 0.305 | 0.163 | 0.155 | 0.140 | 0.327 | 0.293 | 0.314 |
| | 20 | 0.367 | 0.362 | 0.334 | 0.221 | 0.212 | 0.180 | 0.357 | 0.333 | 0.321 |
| | 50 | 0.428 | 0.414 | 0.380 | 0.208 | 0.251 | 0.232 | 0.399 | 0.391 | 0.372 |

| T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $H_{3m}(\mathcal{K}^*)$ | $\mathcal{M}^{1*}_m$ | $\mathcal{M}^{2*}_m$ | $\mathcal{M}^{3*}_m$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.362 | 0.361 | 0.334 | 0.211 | 0.183 | 0.181 | 0.359 | 0.345 | 0.337 |
| | 20 | 0.420 | 0.410 | 0.383 | 0.230 | 0.205 | 0.203 | 0.407 | 0.382 | 0.392 |
| | 50 | 0.477 | 0.459 | 0.434 | 0.256 | 0.215 | 0.246 | 0.442 | 0.427 | 0.406 |
| 100 | 10 | 0.336 | 0.332 | 0.304 | 0.160 | 0.152 | 0.139 | 0.325 | 0.292 | 0.313 |
| | 20 | 0.365 | 0.360 | 0.334 | 0.221 | 0.211 | 0.180 | 0.356 | 0.332 | 0.320 |
| | 50 | 0.428 | 0.414 | 0.379 | 0.251 | 0.230 | 0.207 | 0.399 | 0.391 | 0.371 |

**Notes :** Empirical sizes corresponding to a 5% nominal size.

Table 8: Size of DeFactored Panel Tests. Intercept case

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 10 | 0.067 | 0.065 | 0.066 | 0.080 | 0.086 | 0.072 | 0.061 | 0.073 | 0.061 |
| | 20 | 0.070 | 0.079 | 0.069 | 0.081 | 0.092 | 0.075 | 0.066 | 0.069 | 0.055 |
| | 50 | 0.061 | 0.052 | 0.064 | 0.100 | 0.110 | 0.096 | 0.072 | 0.069 | 0.070 |
| 100 | 10 | 0.065 | 0.071 | 0.064 | 0.085 | 0.099 | 0.089 | 0.085 | 0.081 | 0.085 |
| | 20 | 0.078 | 0.072 | 0.071 | 0.096 | 0.102 | 0.089 | 0.075 | 0.077 | 0.077 |
| | 50 | 0.088 | 0.080 | 0.075 | 0.095 | 0.111 | 0.095 | 0.091 | 0.096 | 0.094 |
| T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
| 50 | 10 | 0.071 | 0.067 | 0.070 | 0.086 | 0.089 | 0.075 | 0.067 | 0.079 | 0.063 |
| | 20 | 0.073 | 0.079 | 0.070 | 0.084 | 0.092 | 0.077 | 0.068 | 0.072 | 0.062 |
| | 50 | 0.061 | 0.053 | 0.066 | 0.102 | 0.110 | 0.097 | 0.073 | 0.071 | 0.073 |
| 100 | 10 | 0.067 | 0.072 | 0.067 | 0.086 | 0.099 | 0.091 | 0.087 | 0.082 | 0.086 |
| | 20 | 0.080 | 0.072 | 0.071 | 0.096 | 0.104 | 0.091 | 0.076 | 0.079 | 0.079 |
| | 50 | 0.090 | 0.080 | 0.076 | 0.096 | 0.111 | 0.095 | 0.093 | 0.096 | 0.095 |
| T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $H_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_m^{3*}$ |
| 50 | 10 | 0.068 | 0.067 | 0.069 | 0.084 | 0.088 | 0.073 | 0.067 | 0.078 | 0.062 |
| | 20 | 0.071 | 0.077 | 0.070 | 0.083 | 0.092 | 0.075 | 0.067 | 0.072 | 0.059 |
| | 50 | 0.061 | 0.053 | 0.066 | 0.100 | 0.110 | 0.096 | 0.073 | 0.071 | 0.073 |
| 100 | 10 | 0.066 | 0.072 | 0.064 | 0.086 | 0.099 | 0.089 | 0.085 | 0.082 | 0.086 |
| | 20 | 0.080 | 0.072 | 0.071 | 0.096 | 0.102 | 0.089 | 0.076 | 0.078 | 0.077 |
| | 50 | 0.088 | 0.080 | 0.076 | 0.095 | 0.111 | 0.095 | 0.092 | 0.096 | 0.094 |

**Notes :**Empirical sizes corresponding to a 5% nominal size.

Table 9: Power of Defactored Panel Tests. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^1$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 10 | 0.124 | 0.133 | 0.129 | 0.098 | 0.089 | 0.091 | 0.124 | 0.118 | 0.122 |
| | | | 20 | 0.129 | 0.148 | 0.135 | 0.095 | 0.093 | 0.095 | 0.115 | 0.129 | 0.124 |
| | | | 50 | 0.189 | 0.206 | 0.201 | 0.114 | 0.098 | 0.103 | 0.159 | 0.176 | 0.170 |
| | | 100 | 10 | 0.388 | 0.380 | 0.394 | 0.176 | 0.130 | 0.193 | 0.357 | 0...361 | 0.382 |
| | | | 20 | 0.460 | 0.494 | 0.495 | 0.203 | 0.142 | 0.230 | 0.469 | 0.469 | 0.512 |
| | | | 50 | 0.752 | 0.756 | 0.746 | 0.170 | 0.352 | 0.310 | 0.741 | 0.735 | 0.768 |
| 0.5 | | 50 | 10 | 0.122 | 0.134 | 0.124 | 0.066 | 0.070 | 0.061 | 0.093 | 0.104 | 0.095 |
| | | | 20 | 0.121 | 0.149 | 0.130 | 0.066 | 0.074 | 0.062 | 0.098 | 0.113 | 0.093 |
| | | | 50 | 0.185 | 0.234 | 0.198 | 0.082 | 0.081 | 0.079 | 0.110 | 0.131 | 0.111 |
| | | 100 | 10 | 0.352 | 0.417 | 0.364 | 0.061 | 0.054 | 0.075 | 0.250 | 0...304 | 0.276 |
| | | | 20 | 0.432 | 0.523 | 0.450 | 0.064 | 0.041 | 0.068 | 0.284 | 0.341 | 0.316 |
| | | | 50 | 0.719 | 0.797 | 0.745 | 0.054 | 0.041 | 0.058 | 0.521 | 0.599 | 0.561 |
| 0.7 | | 50 | 10 | 0.097 | 0.102 | 0.100 | 0.072 | 0.076 | 0.069 | 0.073 | 0.089 | 0.078 |
| | | | 20 | 0.089 | 0.111 | 0.097 | 0.068 | 0.078 | 0.063 | 0.076 | 0.088 | 0.076 |
| | | | 50 | 0.110 | 0.139 | 0.111 | 0.077 | 0.087 | 0.080 | 0.088 | 0.103 | 0.092 |
| | | 100 | 10 | 0.190 | 0.258 | 0.202 | 0.056 | 0.062 | 0.064 | 0.143 | 0.197 | 0.151 |
| | | | 20 | 0.235 | 0.332 | 0.249 | 0.048 | 0.048 | 0.054 | 0.145 | 0.216 | 0.156 |
| | | | 50 | 0.405 | 0.554 | 0.415 | 0.054 | 0.043 | 0.055 | 0.211 | 0.340 | 0.243 |
| 0.3 | 0.10 | 50 | 10 | 0.353 | 0.361 | 0.355 | 0.168 | 0.122 | 0.176 | 0.351 | 0.335 | 0.361 |
| | | | 20 | 0.475 | 0.473 | 0.480 | 0.197 | 0.127 | 0.204 | 0.457 | 0.430 | 0.461 |
| | | | 50 | 0.713 | 0.729 | 0.723 | 0.295 | 0.168 | 0.318 | 0.703 | 0.702 | 0.727 |
| | | 100 | 10 | 0.873 | 0.852 | 0.871 | 0.552 | 0.311 | 0.568 | 0.900 | 0...876 | 0.909 |
| | | | 20 | 0.944 | 0.924 | 0.943 | 0.679 | 0.410 | 0.712 | 0.959 | 0.948 | 0.964 |
| | | | 50 | 0.951 | 0.946 | 0.953 | 0.876 | 0.580 | 0.903 | 0.973 | 0.963 | 0.979 |
| 0.5 | | 50 | 10 | 0.325 | 0.390 | 0.334 | 0.057 | 0.044 | 0.056 | 0.232 | 0.268 | 0.246 |
| | | | 20 | 0.423 | 0.509 | 0.436 | 0.067 | 0.050 | 0.067 | 0.280 | 0.350 | 0.301 |
| | | | 50 | 0.668 | 0.752 | 0.689 | 0.062 | 0.036 | 0.064 | 0.473 | 0.555 | 0.500 |
| | | 100 | 10 | 0.845 | 0.875 | 0.856 | 0.129 | 0.044 | 0.151 | 0.768 | 0...817 | 0.788 |
| | | | 20 | 0.916 | 0.942 | 0.922 | 0.116 | 0.025 | 0.136 | 0.878 | 0.905 | 0.883 |
| | | | 50 | 0.951 | 0.946 | 0.945 | 0.132 | 0.025 | 0.167 | 0.950 | 0.944 | 0.946 |
| 0.7 | | 50 | 10 | 0.197 | 0.253 | 0.195 | 0.050 | 0.052 | 0.049 | 0.124 | 0.159 | 0.135 |
| | | | 20 | 0.221 | 0.303 | 0.236 | 0.053 | 0.056 | 0.051 | 0.127 | 0.187 | 0.135 |
| | | | 50 | 0.383 | 0.513 | 0.387 | 0.054 | 0.057 | 0.051 | 0.189 | 0.303 | 0.209 |
| | | 100 | 10 | 0.592 | 0.728 | 0.613 | 0.031 | 0.030 | 0.038 | 0.440 | 0.603 | 0.472 |
| | | | 20 | 0.734 | 0.847 | 0.753 | 0.021 | 0.019 | 0.022 | 0.557 | 0.730 | 0.604 |
| | | | 50 | 0.900 | 0.919 | 0.903 | 0.028 | 0.026 | 0.029 | 0.802 | 0.899 | 0.841 |

Table 9: continued.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 10 | 0.919 | 0.899 | 0.921 | 0.624 | 0.387 | 0.633 | 0.926 | 0.913 | 0.929 |
| | | | 20 | 0.959 | 0.945 | 0.958 | 0.753 | 0.476 | 0.760 | 0.973 | 0.952 | 0.971 |
| | | | 50 | 0.978 | 0.968 | 0.980 | 0.916 | 0.703 | 0.922 | 0.988 | 0.980 | 0.988 |
| | | 100 | 10 | 0.984 | 0.982 | 0.984 | 0.869 | 0.666 | 0.881 | 0.995 | 0...991 | 0.994 |
| | | | 20 | 0.988 | 0.988 | 0.988 | 0.956 | 0.831 | 0.961 | 0.998 | 0.998 | 0.995 |
| | | | 50 | 0.992 | 0.991 | 0.992 | 0.966 | 0.932 | 0.967 | 0.997 | 0.996 | 0.998 |
| 0.5 | | 50 | 10 | 0.896 | 0.912 | 0.906 | 0.159 | 0.032 | 0.169 | 0.848 | 0.872 | 0.849 |
| | | | 20 | 0.949 | 0.950 | 0.947 | 0.176 | 0.021 | 0.198 | 0.922 | 0.920 | 0.925 |
| | | | 50 | 0.964 | 0.964 | 0.964 | 0.224 | 0.019 | 0.267 | 0.959 | 0.946 | 0.956 |
| | | 100 | 10 | 0.982 | 0.985 | 0.982 | 0.408 | 0.070 | 0.443 | 0.982 | 0...986 | 0.984 |
| | | | 20 | 0.987 | 0.989 | 0.987 | 0.535 | 0.058 | 0.587 | 0.989 | 0.995 | 0.991 |
| | | | 50 | 0.990 | 0.990 | 0.990 | 0.750 | 0.045 | 0.790 | 0.992 | 0.993 | 0.994 |
| 0.7 | | 50 | 10 | 0.708 | 0.813 | 0.725 | 0.023 | 0.016 | 0.025 | 0.547 | 0.706 | 0.569 |
| | | | 20 | 0.852 | 0.896 | 0.859 | 0.015 | 0.015 | 0.018 | 0.698 | 0.833 | 0.717 |
| | | | 50 | 0.920 | 0.923 | 0.919 | 0.018 | 0.015 | 0.014 | 0.887 | 0.903 | 0.884 |
| | | 100 | 10 | 0.957 | 0.965 | 0.955 | 0.009 | 0.009 | 0.012 | 0.943 | 0.959 | 0.945 |
| | | | 20 | 0.971 | 0.978 | 0.968 | 0.008 | 0.009 | 0.008 | 0.960 | 0.970 | 0.961 |
| | | | 50 | 0.976 | 0.982 | 0.977 | 0.013 | 0.011 | 0.014 | 0.969 | 0.972 | 0.972 |
| 0.3 | 0.50 | 50 | 10 | 0.988 | 0.980 | 0.988 | 0.776 | 0.552 | 0.788 | 0.982 | 0.991 | 0.991 |
| | | | 20 | 0.991 | 0.989 | 0.991 | 0.897 | 0.687 | 0.905 | 0.994 | 0.993 | 0.994 |
| | | | 50 | 1.000 | 0.998 | 1.000 | 0.954 | 0.855 | 0.956 | 1.000 | 1.000 | 1.000 |
| | | 100 | 10 | 0.992 | 0.992 | 0.992 | 0.830 | 0.616 | 0.853 | 0.997 | 0...998 | 0.998 |
| | | | 20 | 0.991 | 0.991 | 0.991 | 0.908 | 0.718 | 0.919 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.965 | 0.854 | 0.970 | 0.996 | 0.998 | 0.998 |
| 0.5 | | 50 | 10 | 0.987 | 0.984 | 0.985 | 0.328 | 0.058 | 0.352 | 0.986 | 0.981 | 0.984 |
| | | | 20 | 0.993 | 0.993 | 0.993 | 0.443 | 0.044 | 0.480 | 0.992 | 0.988 | 0.992 |
| | | | 50 | 0.996 | 0.997 | 0.996 | 0.650 | 0.042 | 0.693 | 0.993 | 0.993 | 0.993 |
| | | 100 | 10 | 0.993 | 0.992 | 0.992 | 0.467 | 0.108 | 0.496 | 0.996 | 0...998 | 0.996 |
| | | | 20 | 0.992 | 0.994 | 0.992 | 0.613 | 0.114 | 0.673 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.815 | 0.153 | 0.853 | 0.996 | 0.996 | 0.996 |
| 0.7 | | 50 | 10 | 0.945 | 0.958 | 0.945 | 0.009 | 0.004 | 0.012 | 0.914 | 0.933 | 0.922 |
| | | | 20 | 0.963 | 0.970 | 0.960 | 0.006 | 0.003 | 0.007 | 0.934 | 0.944 | 0.936 |
| | | | 50 | 0.975 | 0.972 | 0.970 | 0.007 | 0.005 | 0.007 | 0.960 | 0.960 | 0.958 |
| | | 100 | 10 | 0.990 | 0.991 | 0.989 | 0.007 | 0.006 | 0.007 | 0.990 | 0.991 | 0.991 |
| | | | 20 | 0.990 | 0.994 | 0.991 | 0.007 | 0.005 | 0.010 | 0.989 | 0.993 | 0.990 |
| | | | 50 | 0.994 | 0.995 | 0.995 | 0.008 | 0.006 | 0.009 | 0.996 | 0.995 | 0.997 |

Table 10: Power of Defactored Modified Panel Tests. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\mathrm{MH}_1^d(\mathcal{K})$ | $\mathrm{MH}_2^d(\mathcal{K})$ | $\mathrm{MH}_3^d(\mathcal{K})$ | $\mathrm{MH}_1^d(\mathcal{K}^*)$ | $\mathrm{MH}_2^d(\mathcal{K}^*)$ | $\mathrm{MH}_3^d(\mathcal{K}^*)$ | $\mathrm{M}\mathcal{M}^{1*}$ | $\mathrm{M}\mathcal{M}^{2*}$ | $\mathrm{M}\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 10 | 0.126 | 0.135 | 0.131 | 0.101 | 0.091 | 0.094 | 0.134 | 0.128 | 0.130 |
| | | | 20 | 0.132 | 0.151 | 0.139 | 0.099 | 0.094 | 0.099 | 0.120 | 0.132 | 0.125 |
| | | | 50 | 0.192 | 0.206 | 0.202 | 0.116 | 0.098 | 0.104 | 0.161 | 0.179 | 0.173 |
| | | 100 | 10 | 0.393 | 0.384 | 0.403 | 0.180 | 0.131 | 0.195 | 0.365 | 0...368 | 0.388 |
| | | | 20 | 0.465 | 0.497 | 0.498 | 0.204 | 0.144 | 0.233 | 0.475 | 0.481 | 0.516 |
| | | | 50 | 0.747 | 0.753 | 0.760 | 0.312 | 0.171 | 0.354 | 0.743 | 0.738 | 0.770 |
| 0.5 | | 50 | 10 | 0.127 | 0.140 | 0.128 | 0.067 | 0.075 | 0.063 | 0.104 | 0.106 | 0.104 |
| | | | 20 | 0.124 | 0.153 | 0.131 | 0.068 | 0.075 | 0.066 | 0.099 | 0.119 | 0.098 |
| | | | 50 | 0.187 | 0.235 | 0.199 | 0.082 | 0.083 | 0.079 | 0.113 | 0.131 | 0.113 |
| | | 100 | 10 | 0.354 | 0.425 | 0.371 | 0.062 | 0.056 | 0.075 | 0.258 | 0...305 | 0.280 |
| | | | 20 | 0.438 | 0.523 | 0.454 | 0.064 | 0.041 | 0.070 | 0.286 | 0.342 | 0.318 |
| | | | 50 | 0.719 | 0.798 | 0.745 | 0.054 | 0.041 | 0.058 | 0.522 | 0.600 | 0.563 |
| 0.7 | | 50 | 10 | 0.098 | 0.105 | 0.103 | 0.076 | 0.079 | 0.069 | 0.081 | 0.090 | 0.083 |
| | | | 20 | 0.091 | 0.114 | 0.097 | 0.068 | 0.079 | 0.065 | 0.079 | 0.090 | 0.079 |
| | | | 50 | 0.113 | 0.140 | 0.114 | 0.078 | 0.087 | 0.081 | 0.088 | 0.109 | 0.093 |
| | | 100 | 10 | 0.195 | 0.259 | 0.208 | 0.059 | 0.063 | 0.066 | 0.147 | 0.201 | 0.156 |
| | | | 20 | 0.235 | 0.334 | 0.253 | 0.049 | 0.049 | 0.054 | 0.149 | 0.218 | 0.158 |
| | | | 50 | 0.407 | 0.554 | 0.418 | 0.054 | 0.043 | 0.056 | 0.214 | 0.341 | 0.244 |
| 0.3 | 0.10 | 50 | 10 | 0.361 | 0.363 | 0.365 | 0.178 | 0.128 | 0.182 | 0.377 | 0.346 | 0.378 |
| | | | 20 | 0.480 | 0.474 | 0.491 | 0.201 | 0.128 | 0.212 | 0.461 | 0.437 | 0.467 |
| | | | 50 | 0.714 | 0.732 | 0.725 | 0.299 | 0.170 | 0.319 | 0.707 | 0.709 | 0.728 |
| | | 100 | 10 | 0.874 | 0.854 | 0.887 | 0.555 | 0.318 | 0.572 | 0.905 | 0...879 | 0.912 |
| | | | 20 | 0.944 | 0.925 | 0.943 | 0.685 | 0.414 | 0.715 | 0.960 | 0.949 | 0.965 |
| | | | 50 | 0.951 | 0.946 | 0.954 | 0.878 | 0.583 | 0.903 | 0.974 | 0.963 | 0.979 |
| 0.5 | | 50 | 10 | 0.332 | 0.399 | 0.341 | 0.057 | 0.048 | 0.061 | 0.245 | 0.278 | 0.252 |
| | | | 20 | 0.429 | 0.515 | 0.437 | 0.068 | 0.053 | 0.070 | 0.291 | 0.355 | 0.303 |
| | | | 50 | 0.672 | 0.752 | 0.691 | 0.062 | 0.037 | 0.065 | 0.479 | 0.558 | 0.503 |
| | | 100 | 10 | 0.850 | 0.876 | 0.860 | 0.133 | 0.044 | 0.154 | 0.777 | 0...819 | 0.796 |
| | | | 20 | 0.916 | 0.942 | 0.922 | 0.120 | 0.026 | 0.141 | 0.880 | 0.906 | 0.886 |
| | | | 50 | 0.951 | 0.946 | 0.945 | 0.132 | 0.025 | 0.167 | 0.950 | 0.945 | 0.946 |
| 0.7 | | 50 | 10 | 0.206 | 0.259 | 0.200 | 0.053 | 0.054 | 0.050 | 0.132 | 0.165 | 0.138 |
| | | | 20 | 0.225 | 0.310 | 0.240 | 0.053 | 0.056 | 0.051 | 0.131 | 0.188 | 0.139 |
| | | | 50 | 0.388 | 0.513 | 0.390 | 0.055 | 0.057 | 0.051 | 0.194 | 0.311 | 0.215 |
| | | 100 | 10 | 0.596 | 0.731 | 0.620 | 0.031 | 0.030 | 0.038 | 0.448 | 0.613 | 0.481 |
| | | | 20 | 0.736 | 0.847 | 0.756 | 0.021 | 0.019 | 0.023 | 0.562 | 0.733 | 0.609 |
| | | | 50 | 0.900 | 0.919 | 0.905 | 0.029 | 0.026 | 0.029 | 0.804 | 0.900 | 0.842 |

Table 10: continued.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}^d_1(\mathcal{K})$ | $\text{MH}^d_2(\mathcal{K})$ | $\text{MH}^d_3(\mathcal{K})$ | $\text{MH}^d_1(\mathcal{K}^*)$ | $\text{MH}^d_2(\mathcal{K}^*)$ | $\text{MH}^d_3(\mathcal{K}^*)$ | $\text{M}\mathcal{M}^{1*}$ | $\text{M}\mathcal{M}^{2*}$ | $\text{M}\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 10 | 0.921 | 0.902 | 0.924 | 0.636 | 0.392 | 0.649 | 0.929 | 0.915 | 0.931 |
| | | | 20 | 0.960 | 0.946 | 0.958 | 0.755 | 0.479 | 0.768 | 0.973 | 0.954 | 0.972 |
| | | | 50 | 0.978 | 0.968 | 0.980 | 0.916 | 0.704 | 0.922 | 0.989 | 0.980 | 0.988 |
| | | 100 | 10 | 0.984 | 0.982 | 0.986 | 0.871 | 0.669 | 0.883 | 0.995 | 0...991 | 0.994 |
| | | | 20 | 0.988 | 0.988 | 0.988 | 0.957 | 0.832 | 0.962 | 0.995 | 0.998 | 0.995 |
| | | | 50 | 0.992 | 0.991 | 0.992 | 0.966 | 0.932 | 0.967 | 0.998 | 0.996 | 0.998 |
| 0.5 | | 50 | 10 | 0.897 | 0.912 | 0.906 | 0.164 | 0.034 | 0.172 | 0.854 | 0.881 | 0.862 |
| | | | 20 | 0.949 | 0.952 | 0.948 | 0.178 | 0.021 | 0.200 | 0.922 | 0.921 | 0.926 |
| | | | 50 | 0.964 | 0.964 | 0.964 | 0.019 | 0.272 | 0.229 | 0.946 | 0.958 | 0.959 |
| | | 100 | 10 | 0.982 | 0.985 | 0.985 | 0.417 | 0.070 | 0.453 | 0.982 | 0...987 | 0.984 |
| | | | 20 | 0.987 | 0.989 | 0.987 | 0.541 | 0.058 | 0.590 | 0.990 | 0.995 | 0.991 |
| | | | 50 | 0.990 | 0.990 | 0.990 | 0.751 | 0.045 | 0.792 | 0.993 | 0.993 | 0.994 |
| 0.7 | | 50 | 10 | 0.712 | 0.815 | 0.729 | 0.023 | 0.017 | 0.026 | 0.563 | 0.718 | 0.583 |
| | | | 20 | 0.857 | 0.896 | 0.862 | 0.016 | 0.015 | 0.019 | 0.707 | 0.835 | 0.731 |
| | | | 50 | 0.920 | 0.923 | 0.919 | 0.018 | 0.015 | 0.014 | 0.877 | 0.903 | 0.886 |
| | | 100 | 10 | 0.958 | 0.965 | 0.958 | 0.010 | 0.009 | 0.012 | 0.944 | 0.959 | 0.945 |
| | | | 20 | 0.971 | 0.978 | 0.968 | 0.009 | 0.009 | 0.009 | 0.960 | 0.970 | 0.961 |
| | | | 50 | 0.976 | 0.982 | 0.977 | 0.013 | 0.011 | 0.014 | 0.970 | 0.972 | 0.972 |
| 0.3 | 0.50 | 50 | 10 | 0.988 | 0.981 | 0.989 | 0.785 | 0.566 | 0.793 | 0.991 | 0.983 | 0.991 |
| | | | 20 | 0.992 | 0.989 | 0.991 | 0.899 | 0.693 | 0.908 | 0.994 | 0.993 | 0.994 |
| | | | 50 | 1.000 | 0.998 | 1.000 | 0.954 | 0.855 | 0.958 | 1.000 | 1.000 | 1.000 |
| | | 100 | 10 | 0.992 | 0.992 | 0.994 | 0.838 | 0.625 | 0.856 | 0.997 | 0...998 | 0.998 |
| | | | 20 | 0.991 | 0.992 | 0.991 | 0.910 | 0.726 | 0.921 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.966 | 0.855 | 0.970 | 0.997 | 0.999 | 0.998 |
| 0.5 | | 50 | 10 | 0.987 | 0.984 | 0.988 | 0.336 | 0.059 | 0.358 | 0.986 | 0.981 | 0.984 |
| | | | 20 | 0.993 | 0.993 | 0.993 | 0.456 | 0.046 | 0.486 | 0.993 | 0.988 | 0.992 |
| | | | 50 | 0.996 | 0.997 | 0.996 | 0.651 | 0.043 | 0.699 | 0.993 | 0.993 | 0.993 |
| | | 100 | 10 | 0.992 | 0.993 | 0.994 | 0.479 | 0.087 | 0.501 | 0.996 | 0...998 | 0.996 |
| | | | 20 | 0.992 | 0.994 | 0.992 | 0.620 | 0.116 | 0.676 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.817 | 0.153 | 0.854 | 0.996 | 0.996 | 0.996 |
| 0.7 | | 50 | 10 | 0.948 | 0.959 | 0.946 | 0.009 | 0.004 | 0.012 | 0.919 | 0.935 | 0.922 |
| | | | 20 | 0.965 | 0.970 | 0.960 | 0.007 | 0.004 | 0.008 | 0.937 | 0.944 | 0.936 |
| | | | 50 | 0.975 | 0.972 | 0.970 | 0.007 | 0.005 | 0.007 | 0.960 | 0.960 | 0.960 |
| | | 100 | 10 | 0.990 | 0.991 | 0.991 | 0.007 | 0.006 | 0.008 | 0.990 | 0.991 | 0.991 |
| | | | 20 | 0.990 | 0.994 | 0.991 | 0.007 | 0.005 | 0.010 | 0.989 | 0.993 | 0.990 |
| | | | 50 | 0.994 | 0.995 | 0.995 | 0.008 | 0.006 | 0.009 | 0.996 | 0.995 | 0.997 |

Table 11: Power of Defactored Modified Min Panel Tests. Intercept case.

| $\tau$ | $\sigma\eta$ | T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_{\mathrm{m}}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.05 | 50 | 10 | 0.126 | 0.135 | 0.131 | 0.100 | 0.091 | 0.094 | 0.132 | 0.125 | 0.127 |
| | | | 20 | 0.132 | 0.150 | 0.137 | 0.097 | 0.094 | 0.096 | 0.119 | 0.132 | 0.124 |
| | | | 50 | 0.191 | 0.206 | 0.202 | 0.116 | 0.098 | 0.103 | 0.159 | 0.178 | 0.172 |
| | | 100 | 10 | 0.390 | 0.380 | 0.396 | 0.178 | 0.130 | 0.194 | 0.361 | 0...364 | 0.384 |
| | | | 20 | 0.462 | 0.494 | 0.495 | 0.203 | 0.142 | 0.230 | 0.471 | 0.471 | 0.512 |
| | | | 50 | 0.746 | 0.752 | 0.758 | 0.311 | 0.171 | 0.352 | 0.742 | 0.736 | 0.769 |
| 0.5 | | 50 | 10 | 0.126 | 0.139 | 0.126 | 0.067 | 0.074 | 0.063 | 0.101 | 0.106 | 0.103 |
| | | | 20 | 0.122 | 0.153 | 0.130 | 0.066 | 0.075 | 0.064 | 0.099 | 0.118 | 0.096 |
| | | | 50 | 0.187 | 0.235 | 0.199 | 0.082 | 0.082 | 0.079 | 0.113 | 0.131 | 0.112 |
| | | 100 | 10 | 0.352 | 0.420 | 0.365 | 0.061 | 0.054 | 0.075 | 0.252 | 0...304 | 0.278 |
| | | | 20 | 0.435 | 0.523 | 0.451 | 0.064 | 0.041 | 0.068 | 0.284 | 0.341 | 0.317 |
| | | | 50 | 0.719 | 0.798 | 0.745 | 0.054 | 0.041 | 0.058 | 0.521 | 0.600 | 0.562 |
| 0.7 | | 50 | 10 | 0.101 | 0.098 | 0.104 | 0.069 | 0.076 | 0.077 | 0.079 | 0.090 | 0.082 |
| | | | 20 | 0.090 | 0.113 | 0.097 | 0.068 | 0.079 | 0.063 | 0.078 | 0.090 | 0.076 |
| | | | 50 | 0.111 | 0.140 | 0.113 | 0.078 | 0.087 | 0.080 | 0.088 | 0.108 | 0.093 |
| | | 100 | 10 | 0.191 | 0.259 | 0.203 | 0.057 | 0.062 | 0.065 | 0.145 | 0.198 | 0.155 |
| | | | 20 | 0.238 | 0.332 | 0.250 | 0.049 | 0.048 | 0.054 | 0.148 | 0.217 | 0.157 |
| | | | 50 | 0.406 | 0.554 | 0.416 | 0.054 | 0.043 | 0.055 | 0.212 | 0.340 | 0.244 |
| 0.3 | 0.10 | 50 | 10 | 0.357 | 0.363 | 0.359 | 0.173 | 0.128 | 0.177 | 0.364 | 0.345 | 0.369 |
| | | | 20 | 0.478 | 0.474 | 0.485 | 0.197 | 0.128 | 0.209 | 0.459 | 0.435 | 0.464 |
| | | | 50 | 0.714 | 0.730 | 0.723 | 0.297 | 0.169 | 0.318 | 0.707 | 0.707 | 0.727 |
| | | 100 | 10 | 0.874 | 0.853 | 0.871 | 0.552 | 0.313 | 0.568 | 0.900 | 0...876 | 0.910 |
| | | | 20 | 0.944 | 0.924 | 0.943 | 0.681 | 0.410 | 0.713 | 0.959 | 0.948 | 0.965 |
| | | | 50 | 0.951 | 0.946 | 0.954 | 0.878 | 0.583 | 0.903 | 0.973 | 0.963 | 0.979 |
| 0.5 | | 50 | 10 | 0.330 | 0.396 | 0.338 | 0.057 | 0.047 | 0.059 | 0.240 | 0.275 | 0.251 |
| | | | 20 | 0.425 | 0.513 | 0.436 | 0.068 | 0.052 | 0.068 | 0.289 | 0.354 | 0.303 |
| | | | 50 | 0.670 | 0.752 | 0.691 | 0.062 | 0.036 | 0.065 | 0.477 | 0.556 | 0.503 |
| | | 100 | 10 | 0.847 | 0.875 | 0.857 | 0.131 | 0.044 | 0.152 | 0.770 | 0...817 | 0.789 |
| | | | 20 | 0.916 | 0.942 | 0.922 | 0.116 | 0.025 | 0.136 | 0.879 | 0.905 | 0.884 |
| | | | 50 | 0.951 | 0.946 | 0.945 | 0.132 | 0.025 | 0.167 | 0.950 | 0.945 | 0.946 |
| 0.7 | | 50 | 10 | 0.205 | 0.258 | 0.196 | 0.052 | 0.053 | 0.050 | 0.131 | 0.162 | 0.137 |
| | | | 20 | 0.225 | 0.308 | 0.238 | 0.053 | 0.056 | 0.051 | 0.129 | 0.188 | 0.137 |
| | | | 50 | 0.386 | 0.517 | 0.390 | 0.055 | 0.057 | 0.051 | 0.194 | 0.310 | 0.214 |
| | | 100 | 10 | 0.594 | 0.729 | 0.614 | 0.031 | 0.030 | 0.038 | 0.444 | 0.608 | 0.474 |
| | | | 20 | 0.734 | 0.847 | 0.754 | 0.021 | 0.019 | 0.023 | 0.559 | 0.732 | 0.605 |
| | | | 50 | 0.900 | 0.919 | 0.903 | 0.029 | 0.026 | 0.029 | 0.803 | 0.900 | 0.841 |

Table 11: continued.

| $\tau$ | $\sigma_\eta$ | T | N | $H_{1m}(\mathcal{K})$ | $H_{2m}(\mathcal{K})$ | $H_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $\mathcal{M}_m^{1*}$ | $\mathcal{M}_m^{2*}$ | $\mathcal{M}_{\mathrm{m}}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.25 | 50 | 10 | 0.920 | 0.901 | 0.921 | 0.629 | 0.388 | 0.638 | 0.928 | 0.915 | 0.930 |
| | | | 20 | 0.960 | 0.946 | 0.958 | 0.753 | 0.477 | 0.761 | 0.973 | 0.954 | 0.971 |
| | | | 50 | 0.978 | 0.968 | 0.980 | 0.916 | 0.704 | 0.922 | 0.989 | 0.980 | 0.988 |
| | | 100 | 10 | 0.984 | 0.982 | 0.984 | 0.870 | 0.667 | 0.881 | 0.995 | 0...991 | 0.994 |
| | | | 20 | 0.988 | 0.988 | 0.988 | 0.956 | 0.831 | 0.961 | 0.995 | 0.995 | 0.995 |
| | | | 50 | 0.992 | 0.992 | 0.992 | 0.966 | 0.932 | 0.967 | 0.997 | 0.996 | 0.998 |
| 0.5 | | 50 | 10 | 0.896 | 0.912 | 0.906 | 0.160 | 0.033 | 0.169 | 0.851 | 0.878 | 0.853 |
| | | | 20 | 0.949 | 0.952 | 0.947 | 0.177 | 0.021 | 0.199 | 0.922 | 0.921 | 0.926 |
| | | | 50 | 0.964 | 0.964 | 0.964 | 0.229 | 0.019 | 0.272 | 0.959 | 0.946 | 0.958 |
| | | 100 | 10 | 0.982 | 0.985 | 0.982 | 0.408 | 0.070 | 0.443 | 0.982 | 0...986 | 0.984 |
| | | | 20 | 0.987 | 0.989 | 0.987 | 0.535 | 0.058 | 0.587 | 0.989 | 0.995 | 0.991 |
| | | | 50 | 0.990 | 0.990 | 0.990 | 0.751 | 0.045 | 0.790 | 0.992 | 0.993 | 0.994 |
| 0.7 | | 50 | 10 | 0.710 | 0.814 | 0.727 | 0.023 | 0.017 | 0.025 | 0.555 | 0.716 | 0.573 |
| | | | 20 | 0.855 | 0.896 | 0.859 | 0.016 | 0.015 | 0.018 | 0.702 | 0.835 | 0.727 |
| | | | 50 | 0.923 | 0.920 | 0.919 | 0.018 | 0.015 | 0.014 | 0.877 | 0.903 | 0.886 |
| | | 100 | 10 | 0.957 | 0.965 | 0.955 | 0.009 | 0.009 | 0.012 | 0.944 | 0.959 | 0.945 |
| | | | 20 | 0.971 | 0.978 | 0.968 | 0.008 | 0.009 | 0.008 | 0.960 | 0.970 | 0.961 |
| | | | 50 | 0.976 | 0.982 | 0.977 | 0.013 | 0.011 | 0.014 | 0.969 | 0.972 | 0.972 |
| 0.3 | 0.50 | 50 | 10 | 0.988 | 0.980 | 0.988 | 0.776 | 0.552 | 0.789 | 0.991 | 0.983 | 0.991 |
| | | | 20 | 0.992 | 0.989 | 0.991 | 0.898 | 0.693 | 0.905 | 0.994 | 0.993 | 0.994 |
| | | | 50 | 0.998 | 1.000 | 1.000 | 0.954 | 0.855 | 0.957 | 1.000 | 1.000 | 1.000 |
| | | 100 | 10 | 0.992 | 0.992 | 0.992 | 0.832 | 0.617 | 0.853 | 0.997 | 0...998 | 0.998 |
| | | | 20 | 0.991 | 0.992 | 0.991 | 0.908 | 0.718 | 0.919 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.965 | 0.855 | 0.970 | 0.996 | 0.998 | 0.998 |
| 0.5 | | 50 | 10 | 0.987 | 0.984 | 0.985 | 0.333 | 0.059 | 0.353 | 0.986 | 0.981 | 0.984 |
| | | | 20 | 0.993 | 0.993 | 0.993 | 0.447 | 0.046 | 0.480 | 0.993 | 0.988 | 0.992 |
| | | | 50 | 0.996 | 0.997 | 0.996 | 0.650 | 0.042 | 0.694 | 0.993 | 0.993 | 0.993 |
| | | 100 | 10 | 0.992 | 0.993 | 0.992 | 0.467 | 0.084 | 0.498 | 0.996 | 0...998 | 0.996 |
| | | | 20 | 0.992 | 0.994 | 0.992 | 0.614 | 0.114 | 0.674 | 0.996 | 0.999 | 0.996 |
| | | | 50 | 0.994 | 0.994 | 0.994 | 0.815 | 0.153 | 0.853 | 0.996 | 0.996 | 0.996 |
| 0.7 | | 50 | 10 | 0.947 | 0.959 | 0.945 | 0.009 | 0.004 | 0.012 | 0.918 | 0.934 | 0.922 |
| | | | 20 | 0.964 | 0.970 | 0.960 | 0.007 | 0.004 | 0.007 | 0.937 | 0.944 | 0.936 |
| | | | 50 | 0.975 | 0.972 | 0.970 | 0.007 | 0.005 | 0.007 | 0.960 | 0.960 | 0.960 |
| | | 100 | 10 | 0.990 | 0.991 | 0.991 | 0.007 | 0.006 | 0.008 | 0.990 | 0.991 | 0.991 |
| | | | 20 | 0.990 | 0.994 | 0.991 | 0.007 | 0.005 | 0.010 | 0.989 | 0.993 | 0.990 |
| | | | 50 | 0.994 | 0.995 | 0.995 | 0.008 | 0.006 | 0.009 | 0.996 | 0.995 | 0.997 |

Table 12: Inflation rate 1970.1-2006.2

| No cross-sectional dependence adjusted tests | | | | | |
|---|---|---|---|---|---|
| Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) |
| $H_1(\mathcal{K})$ | 1.979 (0.044) | $H_1(\mathcal{K}^*)$ | 390.0 (0.000) | $\mathcal{M}^{1*}$ | 337.3 (0.000) |
| $H_2(\mathcal{K})$ | −0.177 (0.570) | $H_2(\mathcal{K}^*)$ | 209.2 (0.000) | $\mathcal{M}^{2*}$ | 189.1 (0.000) |
| $MH_1^d(\mathcal{K})$ | 2.065 (0.019) | $MH_1^d(\mathcal{K}^*)$ | 396.0 (0.000) | $M\mathcal{M}^{1*}$ | 343.7 (0.000) |
| $MH_2^d(\mathcal{K})$ | −0.135 (0.210) | $MH_2^d(\mathcal{K}^*)$ | 211.7 (0.000) | $M\mathcal{M}^{2*}$ | 191.8 (0.000) |
| $MH_{1_m}(\mathcal{K})$ | 1.983 (0.024) | $MH_{1_m}(\mathcal{K}^*)$ | 390.4 (0.000) | $M\mathcal{M}_m^{1*}$ | 337.7 (0.000) |
| $MH_{2_m}(\mathcal{K})$ | −0.175 (0.569) | $MH_{2_m}(\mathcal{K}^*)$ | 209.2 (0.000) | $M\mathcal{M}_m^{2*}$ | 189.3 (0.000) |
| Break Date : 1978.3 | | | | | |
| Cross-correlation dependence adjusted tests | | | | | |
| Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) |
| $H_1(\mathcal{K})$ | 0.261 (0.397) | $H_1(\mathcal{K}^*)$ | 24.5 (0.000) | $\mathcal{M}^{1*}$ | 19.6 (0.000) |
| $H_2(\mathcal{K})$ | −1.449 (0.926) | $H_2(\mathcal{K}^*)$ | 18.0 (0.000) | $\mathcal{M}^{2*}$ | 14.3 (0.000) |
| $MH_1^d(\mathcal{K})$ | 0.277 (0.391) | $MH_1^d(\mathcal{K}^*)$ | 24.6 (0.000) | $M\mathcal{M}^{1*}$ | 19.7 (0.000) |
| $MH_2^d(\mathcal{K})$ | −1.441 (0.925) | $MH_2^d(\mathcal{K}^*)$ | 18.1 (0.000) | $M\mathcal{M}^{2*}$ | 14.4 (0.000) |
| $MH_{1_m}(\mathcal{K})$ | 0.263 (0.396) | $MH_{1_m}(\mathcal{K}^*)$ | 24.6 (0.000) | $M\mathcal{M}_m^{1*}$ | 19.6 (0.000) |
| $MH_{2_m}(\mathcal{K})$ | −1.448 (0.926) | $MH_{2_m}(\mathcal{K}^*)$ | 18.5 (0.000) | $M\mathcal{M}_m^{2*}$ | 14.3 (0.000) |
| Break Date : 1984.4 | | | | | |