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# Change in persistence tests for panels: an update and some new results

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# Change in persistence tests for panels: an update and some new results.*

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**Abstract**

In this paper we propose a set of new panel tests to detect changes in persistence. The test statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1), from I(1) to I(0) and in an unknown direction. The limiting distributions of the panel tests are derived, and small sample properties are investigated by Monte Carlo experiments under the hypothesis that the individual series are independently cross-section distributed. These tests have a good size and power properties. Cross-sectional dependence is also considered. A procedure of de-factorizing, proposed by Stock and Watson (2002), is applied. The defactored panel tests have good size and power. The empirical results obtained from applying these tests to a panel covering 21 OECD countries observed between 1970 and 2007 suggest that inflation rate changes from I(1) to I(0) when cross-correlation is considered.

**Keywords**: Persistence, Stationarity, Panel data.
**JEL Classification**: C12, C23.

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# 1 Introduction

Recent time series literature has shown that economic and financial data are characterized by a change in persistence between separate I(1) and I(0) regimes rather than simply I(1) or I(0) behavior. For example, Cogley and Sargent (2001) and Emery (1994), using post World War II data, argued that persistence in U.S. inflation has decreased substantially since the early 1980s. Strikingly, Emery (1994) finds that U.S. inflation in the 1980s can best be described as a white noise. Further evidence of change in persistence from I(1) to I(0) behavior in U.S. inflation is also reported in Kim (2000), Busetti and Taylor (2004) and Leybourne et al. (2003). Other variables for which changes in persistence have been observed include real output (e.g Taylor, 2005) and short-term interest rates (e.g. Mankiw *et al.*, 1987)).

A number of testing procedures have been developed to test for changes in persistence. The most popular of these are the ratio-based change in persistence tests of Kim (2000), Kim *et al.* (2002), Busetti and Taylor (2004) and Harvey *et al.* (2006). These test the null hypothesis that a series is a constant I(0) process against the alternative that it displays a change in persistence, either from I(0) to I(1), or viceversa. Kim (2000) and Kim *et al.* (2002) proposed a residual-based ratio test against changes in persistence in a time series, focusing on the case of a shift from I(0) to I(1) at some point in the sample. Kim (2000) also discussed the possibility of I(1) to I(0) shifts but did not provide tests against such an alternative. Busetti and Taylor (2004) proposed new ratio-based tests and breakpoint estimators which are consistent under I(1) to I(0) changes, and they demonstrated that the ratio-based tests which are consistent against changes from I(1) to I(0) are not consistent against changes from I(0) to I(1), and viceversa, with neither consistent against constant I(1) processes. Harvey *et al.* (2006) developed a set of new tests which are based on a modified version of the ratio-base statistics of Kim (2000), Kim *et al.* (2002) and Busetti and Taylor (2004). These modifications use the variable addition approach of Vogelsang (1998), and a recent generalization by Sayginsoy (2003), and yielding tests which, by design, have the same critical values regardless of whether the process is I(0) or (near) I(1) throughout. This technique can only be used with the ratio based test of the null I(0) because other tests of the I(0) (I(1)) null are based on statistics which are divergent under constant I(1) (I(0)) processes. Hence the null hypothesis is constant persistence (either a constant I(0) process or a constant I(1) process), and the alternative is a change in persistence. Using a panel framework, Costantini and Gutierrez (2007) proposed new recursive ADF unit root tests to detect changes in persistence based on the inverse normal Z test suggested by Choi (2001). The small sample properties of the recursive tests are investigated by Monte Carlo experiments. The panel tests have good size and power.

In this paper we propose a set of new panel tests to detect changes in persistence. The test statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1), from I(1) to I(0) and an unknown direction. Two set of panel tests are proposed. The first set is based on the hypothesis that the individual series are independently

cross-section distributed. The second one uses the hypothesis of cross-sectional dependence.
The paper is organized as follows. In section 2 we present a set of new panel tests to detect a change in persistence under the hypothesis of cross-section independence. Section 3 describes the panel tests under the cross-section dependence hypothesis. Section 4 presents Monte Carlo simulations. In section 5 we apply the tests to analyze a panel of 21 OECD inflation rate series for the period 1970.1-2007.3 . Section 6 concludes. The main technical proofs and derivations are in the Appendix.

# 2 Persistence tests without cross-section correlation

## 2.1 The model

Consider the following Gaussian unobserved components model for a sample of $N$ cross-sections observed over $T$ time periods:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + \varepsilon_{i,t}, \qquad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{1}$$

we allow for the following three cases:

- Case 1: I(0) $\rightarrow$ I(1)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [T\tau])\eta_{i,t}, \qquad i = 1, \ldots, N,\ t = 1, \ldots, T, \tag{2}$$

- Case 2: I(1) $\rightarrow$ I(0)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau T])\eta_{i,t} \qquad i = 1, \ldots, N,\ t = 1, \ldots, T. \tag{3}$$

- Case 3: unknown direction I(0) $\rightarrow$ I(1) or I(1) $\rightarrow$ I(0)

where $1(\cdot)$ is the indicator function, $d_{i,t}$ is a deterministic component, $\varepsilon_{i,t}$ and $\eta_{i,t}$ are mutually independent mean zero iid gaussian processes with $\sigma^2_{\varepsilon i}$ and $\sigma^2_{\eta i}$ variance. The deterministic components are taken to be the unity vector.
From (2), it can be easily seen that for each cross section $i$, the data generating process yields a process which is stationary up to and including time $[\tau T]$, with the change-point proportion $\tau \in (0, 1)$, but is I(1) after the break, if and only if, $\sigma^2_{\eta i} > 0$. Otherwise, from (3), we note that for each cross section $i$, the data generating process yields a process which is I(1) up to and including time $[\tau T]$ but it is stationary after the break, if and only if, $\sigma^2_{\eta i} > 0$.
Therefore, the panel test of stationarity against a shift in persistence from stationarity to a unit root or viceversa can be framed in testing the null hypothesis as:

$$\mathbf{H_0} = \sigma^2_{\eta i} = 0, \ \ \forall\, i \tag{4}$$

against the alternative hypothesis

$$\mathbf{H_1} = \sigma^2_{\eta i} > 0, \text{ at least for some } i. \tag{5}$$

When I(0) $\rightarrow$ I(1), we denote the alternative hypothesis as $\mathbf{H_{01}}$. If I(1) $\rightarrow$ I(0), then we use $\mathbf{H_{10}}$.
The following assumption plays a key role in the rest of the paper.

**Assumption 1** *The process* $\{\mu_{i,t}\}_{i,t=0}^{+\infty}$ *is such that for each* $i$

1. $\mathbf{E}[\mu_i] = 0$;
2. $\mathbf{E}|\mu_i|^4 < +\infty$;
3. *fixed* $i$, *then* $\{\mu_{i,t}\}_{t=0}^{+\infty}$ *is* $\phi$*-mixing with mixing coefficients* $\phi_{i,m}$ *such that*
$$\sum_{m=1}^{\infty} \phi_{i,m}^{\gamma_i} < +\infty,$$
*for some* $\gamma_i > 0$;
4. *There exists the long-run variance*
$$\sigma^2_{\mu i} = \sum_{j=0}^{\infty} \mathbf{E}[\mu_{i,j+1}\mu'_{i,1}];$$
5. *for each* $s \in (0,1)$, *we have*
$$\lim_{T\rightarrow\infty} \mathbf{V}\Big[\frac{1}{\sqrt{T}}\sum_{t=1}^{[sT]} \mu_{i,t}\Big] = s\sigma^2_{\mu i}$$
*and*
$$\lim_{T\rightarrow\infty} \mathbf{V}\Big[\frac{1}{\sqrt{T}}\sum_{t=[sT]+1}^{T} \mu_{i,t}\Big] = (1-s)\sigma^2_{\mu i}$$
.

The above conditions have been used by Phillips (1987), Phillips and Perron (1988) and Phillips and Solo (1992), among others, to prove results on the asymptotic distribution of a stochastic process. Finally, note that throughout the next sections we use sequential limits, where $T \rightarrow \infty$ is followed by $N \rightarrow \infty$.

## 2.2 Panel ratio-based tests: I(0) $\rightarrow$ I(1)

In this section we present new panel tests to detect changes in persistence as in (2) and investigate their asymptotic behavior. We show that panel tests are standard normally distributed.
Consider the gaussian process (1)-(2). We want to test the null hypothesis $\mathbf{H_0}$ in (4) against $\mathbf{H_1}$ in (5). Let $\tilde{\varepsilon}_{i,t}$, $i = 1,\ldots,N$ and $t = 1,\ldots,T$, be the residuals from the regression of $y_{i,t}$ on intercept. If a structural change occurs at time $t = [\tau T]$ for $\tau \in (0,1)$, the following partial sum process can be defined as:

$$\begin{cases} S^{(0)}_{i,t} = \sum_{j=1}^{t} \tilde{\varepsilon}_{i,j} & t = 1,\ldots,[T\tau]; \;\; i = 1,\ldots,N, \\ S^{(1)}_{i,t} = \sum_{j=[T\tau]+1}^{t} \tilde{\varepsilon}_{i,j} & t = [T\tau]+1,\ldots,T; \;\; i = 1,\ldots,N, \end{cases} \tag{6}$$

Then, we consider the following test statistic:

$$\mathcal{K}_{T,N}(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \Big[ \frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}{\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2} - \mu \Big], \tag{7}$$

where

$$\mu = \frac{(T-[T\tau])^{-2} \sum_{t=[T\tau]+1}^{T} \mathbf{E}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} \mathbf{E}[S_{i,t}^{(0)}(\tau)^2]} \tag{8}$$

and

$$\sigma = \sqrt{\frac{(T-[T\tau])^{-4} \sum_{t=[T\tau]+1}^{T} \mathbf{V}[S_{i,t}^{(1)}(\tau)^2]}{[T\tau]^{-4} \sum_{t=1}^{[T\tau]} \mathbf{V}[S_{i,t}^{(0)}(\tau)^2]}}. \tag{9}$$

**Theorem 2.1** *Suppose that Assumption 1 is true under the null hypothesis* $\mathbf{H_0}$. *Then*

$$\lim_{N\to+\infty} \lim_{T\to+\infty} \mathcal{K}_{T,N}(\tau) = \mathcal{K}(\tau) \sim N(0,1). \tag{10}$$

In (7), the true value of $\tau$ is unknown. If the the true change period is unknown three transformations of the tests $\mathcal{K}_{T,N}(\tau)$ defined in (7) for testing changes in persistence with unknown break point $[T\tau]$ can be considered:

- A maximum-Chow-type test, as used in Davies (1977), Hawkins (1987), Kim and Siegmund (1989), and Andrews (1993) is

$$\mathrm{H}_1(\mathcal{K}_{T,N}(\tau)) := \sup_{\tau\in(0,1)} \mathcal{K}_{T,N}(\tau). \tag{11}$$

- The mean score test proposed by Hansen (1991)

$$\mathrm{H}_2(\mathcal{K}_{T,N}(\tau)) := \int_{\tau\in(0,1)} \mathcal{K}_{T,N}(\tau) \mathrm{d}\tau. \tag{12}$$

- The mean-exponential test introduced by Andrews and Ploberger (1994), i.e.

$$\mathrm{H}_3(\mathcal{K}_{T,N}(\tau)) := \log\Big\{ \int_{\tau\in(0,1)} \exp[\mathcal{K}_{T,N}(\tau)] \mathrm{d}\tau \Big\}. \tag{13}$$

The asymptotic distribution of the tests defined in (11), (12) and (13) are given in the next result.

**Theorem 2.2** *The following conditions hold true.*

*(i) It results*

$$\lim_{T\to+\infty} \lim_{N\to+\infty} \mathrm{H_j}(\mathcal{K}_{T,N}(\tau)) = \mathrm{H_j}(\mathcal{K}(\tau)), \qquad j = 1,2,3.$$

*(ii) For each* $j = 1,2,3$, *we have* $\mathrm{H_j}(\mathcal{K}(\tau)) \sim N(0,1)$.

## 2.3 Panel reverse test: I(1) → I(0)

Consider the gaussian process (1)-(3). In this case, the null hypothesis refers to the stationary process and the alternative to a shift from I(1) to I(0). The following reverse test statistic is proposed:

$$\mathcal{K}^{\star}_{T,N}(\tau) = \frac{\sqrt{N}}{\sigma} \cdot \Big[ \frac{[T\tau]^{-2}}{(T-[T\tau])^{-2}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}{\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2} - \mu \Big], \tag{14}$$

where

$$\mu = \mathbf{E}\Big[ \frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}{(T-[T\tau])^{-2} \sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2} \Big]$$

and

$$\sigma = \sqrt{\mathbf{V}\Big[ \frac{[T\tau]^{-2} \sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}{(T-[T\tau])^{-2} \sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2} \Big]}.$$

The asymptotic distribution of the statistics defined in (14) is shown by the following result.

**Theorem 2.3** *Suppose that Assumption 1 is true under the null hypothesis* $\mathbf{H_0}$*. Then*

$$\lim_{N\to+\infty} \lim_{T\to+\infty} \mathcal{K}^{\star}_{T,N}(\tau) = \mathcal{K}^{\star}(\tau) \sim N(0,1). \tag{15}$$

In the next result we give the asymptotic distributions of the transformations $\mathrm{H}_1$, $\mathrm{H}_2$ and $\mathrm{H}_3$ of the test $\mathcal{K}^{\star}$.

**Theorem 2.4** *The following propositions hold.*

*(i) It results*

$$\lim_{T\to+\infty} \lim_{N\to+\infty} \mathrm{H_j}(\mathcal{K}^{\star}_{T,N}(\hat{\tau})) = \mathrm{H_j}(\mathcal{K}^{\star}(\tau)), \qquad j=1,2,3.$$

*(ii) For each* $j=1,2,3$, *we have* $\mathrm{H_j}(\mathcal{K}^{\star}(\tau)) \sim N(0,1)$.

## 2.4 Panel tests with unknown direction

We now discuss the case of unknown direction of changes in persistence. Three panel tests are developed and their asymptotic distributions are derived. The tests are:

$$\mathcal{M}^{j,*}_{T,N} = \frac{\sqrt{N}}{\sigma^*_j} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} [\max\{\mathrm{H_j}(\tilde{\mathcal{K}}_{T,i}), \mathrm{H_j}(\tilde{\mathcal{K}}^{*}_{T,i})\} - \mu^*_j], \qquad j=1,2,3; \tag{16}$$

where

$$\tilde{\mathcal{K}}_{T,i} = \frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \frac{\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2},$$

$$\tilde{\mathcal{K}}^{\star}_{T,i} = (\tilde{\mathcal{K}}_{T,i})^{-1},$$

$$\mu_j^* = \mathbf{E}\Big[\max\{\mathrm{H_j}(\tilde{\mathcal{K}}_{\mathrm{T,i}}), \mathrm{H_j}(\tilde{\mathcal{K}}^*_{\mathrm{T,i}})\}\Big] \qquad \mathrm{j} = 1,2,3; \quad \mathrm{i} = 1,\dots,\mathrm{N},$$

$$\sigma_j^* = \sqrt{\mathbf{V}\Big[\max\{\mathrm{H_j}(\tilde{\mathcal{K}}_{\mathrm{T,i}}), \mathrm{H_j}(\tilde{\mathcal{K}}^*_{\mathrm{T,i}})\}\Big]} \qquad j = 1,2,3; \quad i = 1,\dots,N.$$

The asymptotic distributions of these tests are now derived.

**Theorem 2.5** *It results*

$$\lim_{N\to+\infty}\lim_{T\to+\infty} \mathcal{M}^{j,*}_{T,N} \sim N(0,1).$$

## 2.5 Modified panel tests

In this section we propose panel tests that are based on the modified version of the test statistics developed in subsections 2.2-2.4. These tests have the same critical value in the limit as the corresponding unmodified tests under the null hypothesis $\mathbf{H_0}$, and the same limiting critical value is also appropriate under the alternative hypothesis $\mathbf{H_1}$. The modification proposed has no asymptotic effect under the null $\mathbf{H_0}$, so that the limiting distribution of the modified tests is the same of the corresponding unmodified tests. The modified panel tests developed are:

$$\mathrm{MH_j}^d(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{1,N,T}) \cdot \mathrm{H_j}(\mathcal{K}_{T,N}(\tau)), \qquad j = 1,2,3; \tag{17}$$

$$\mathrm{MH_j}^d(\mathcal{K}^\star_{T,N}(\tau)) := \exp(-bJ_{1,N,T}) \cdot \mathrm{H_j}(\mathcal{K}^\star_{T,N}(\tau)), \qquad j = 1,2,3; \tag{18}$$

$$\mathrm{M}\mathcal{M}^{j*}_{T,N} := \exp(-bJ_{1,N,T}) \cdot \mathcal{M}^{j*}_{T,N} \qquad j = 1,2,3; \tag{19}$$

where $b$ is a finite constant and $J_{1,N,T}$ is the arithmetic mean on $N$ of the truncated sequences of $T^{-1}$ times the Wald statistic $J^{(i)}_{1,T}$ for testing the joint hypothesis $\varsigma_{i,k+1} = \dots = \varsigma_{i,9} = 0$ in panel regression

$$y_{i,t} = \varepsilon_{i,t} + \sum_{j=k+1}^{9} \varsigma_{i,j} t^j + \text{error}, \qquad t = 1,\dots,[\tau T]; \;\; i = 1,\dots,N. \tag{20}$$

Under the null hypothesis, Harvey *et al.* (2006) show that

$$\lim_{T\to+\infty} J^{(i)}_{1,T} = 1, \qquad \forall\, i = 1,\dots,N.$$

Therefore, since we assumed independence and identical distribution with respect to the cross-sectional dimension $i$, we have

$$\lim_{T,N\to+\infty} J_{1,N,T} = \lim_{T,N\to+\infty} \frac{1}{N}\sum_{i=1}^{N} J^{(i)}_{1,T} = 1.$$

Consequently, modified panel tests have the same limiting distribution under $\mathbf{H_0}$ as the unmodified tests. Under the alternative hypothesis, using Harvey *et al.*'s (2006) results and the fact that the asymptotic distributions of tests $\mathrm{H_j}(\mathcal{K}_{T,N}(\tau))$, $\mathrm{H_j}(\mathcal{K}^\star_{T,N}(\tau))$, and $\mathcal{M}^{j*}_{T,N}$, $j = 1,2,3$ are standard gaussian (see Theorem 2.2, Theorem 2.4 and Theorem 2.5), for each $j = 1,2,3$, we have

$$\lim_{N\to+\infty} T^{-2}(\mathrm{MH_j}^d(\mathcal{K}_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} (\mathrm{MH_j}^d(\mathcal{K}_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} \{\exp[-bJ_{1,N,T}] - 1\}\mathrm{H_j}(\mathcal{K}_{T,N}(\tau)) = O_p(1)O_p(1) = O_p(1).$$

Analogously, it results

$$\lim_{N\to+\infty} T^{-2}(\mathrm{MH_j}^d(\mathcal{K}^\star_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}^\star_{T,N}(\tau))) = O_p(1)$$

and

$$\lim_{N\to+\infty} T^{-2}(\mathrm{M}\mathcal{M}^{j*}_{T,N} - \mathcal{M}^{j*}_{T,N}) = O_p(1).$$

Thus, we obtain a test which rejects for large values of the modified tests and retains the same rate of consistency under the alternative $\mathbf{H_{01}}$ as the original unmodified $\mathrm{H_j}$ tests. For the alternative $\mathbf{H_{10}}$, the modified tests are also $O_p(1)$. A more appropriate modification procedure is proposed for testing the null against the alternative $\mathbf{H_{01}}$, . Following Harvey *et al.* (2006), we modify tests (17), (18) and (19) by introducing

$$J_{N,min} := \min_{\tau\in(0,1)} J_{1,N,[\tau T]}.$$

We define

$$\mathrm{MH_{j,min}}(\mathcal{K}_{T,N}(\tau)) := \exp(-bJ_{N,min}) \cdot \mathrm{H_j}(\mathcal{K}_{T,N}(\tau)), \qquad j = 1,2,3; \tag{21}$$

$$\mathrm{MH_{j,min}}(\mathcal{K}^\star_{T,N}(\tau)) := \exp(-bJ_{N,min}) \cdot \mathrm{H_j}(\mathcal{K}^\star_{T,N}(\tau)), \qquad j = 1,2,3; \tag{22}$$

$$\mathrm{M}\mathcal{M}^{j*}_{min,T,N} := \exp(-bJ_{N,min}) \cdot \mathcal{M}^{j*}_{T,N} \qquad j = 1,2,3; \tag{23}$$

and

$$J_{min} := \lim_{N\to+\infty} J_{N,min}.$$

in this case also we can derive the asymptotic analysis using the alternative hypothesis of Harvey *et al.* (2006). We have

$$\lim_{N\to+\infty} T^{-2}(\mathrm{MH_{j,min}}(\mathcal{K}_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} (\mathrm{MH_{j,min}}(\mathcal{K}_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}_{T,N}(\tau))) =$$

$$= T^{-2} \lim_{N\to+\infty} \{\exp[-bJ_{N,min}] - 1\}\mathrm{H_j}(\mathcal{K}_{T,N}(\tau)) =$$

$$= T^{-2}\{\exp[-bJ_{min}] - 1\} \lim_{N\to+\infty} \mathrm{H_j}(\mathcal{K}_{T,N}(\tau)) = o_p(1)O_p(1) = o_p(1),$$

and, analogously,

$$\lim_{N\to+\infty} T^{-2}(\mathrm{MH_{j,min}}(\mathcal{K}^\star_{T,N}(\tau)) - \mathrm{H_j}(\mathcal{K}^\star_{T,N}(\tau))) = o_p(1)$$

and

$$\lim_{N\to+\infty} T^{-2}(\mathrm{M}\mathcal{M}^{j*}_{min,T,N} - \mathcal{M}^{j*}_{T,N}) = o_p(1).$$

Thus, the new modification has no asymptotic effect under the alternative $\mathbf{H_{01}}$, unlike to the original $\mathrm{MH_j}^d$ modifications. Under the alternative $\mathbf{H_{10}}$, it is easily shown that the modified min tests are $O_p(1)$.

## 2.6 Estimation of the break

In this section we present a procedure to estimate the unknown change point. Consider the following estimator:

$$\Pi_{N,T}(\tau) = \frac{1}{N}\sum_{i=1}^{N} \frac{\sum_{t=[T\tau]+1}^{T} \tilde{\mu}_{i,t}^2/(T-[T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}_{i,t}^2/[\tau T]}. \tag{24}$$

In order to explore the asymptotic behavior of the estimated unknown change point, the following assumption is necessary.

**Assumption 2** *Let $\tilde{\mu}_{i,s+1}$,$\tilde{\mu}_{i,s+2}$,....., $\tilde{\mu}_{i,s+m}$, for $s \in 0,...,T-1$ and $m \leq T-s$ be a sequence of stationary variables. Assume that $m^{-1}\sum_{t=s+1}^{s+m} \tilde{\mu}_{i,t}^2 \to E[\mu_i^2]$ for $E[\mu_i^2] < \infty$, $\forall i = 1,...,N$.*

Now, let $\hat{\tau}$ be:

$$\hat{\tau} = \left\{ \mathrm{argmax}_{\tau\in(0,1)} \Pi_{\mathrm{N,T}}(\tau) \right\}. \tag{25}$$

The following theorem shows the asymptotic properties of $\hat{\tau}$:

**Theorem 2.6** *Suppose that Assumption 2 holds. Under the alternative hypothesis, it results*

$$(\hat{\tau}-\tau) = o_p(1), \tag{26}$$

$$T(\hat{\tau}-\tau) = O_p(1), \tag{27}$$

# 3 Persistence test with cross-section correlation

Previous testing procedures are valid under the assumption that the units are cross-section independent. However, this requirement is rarely likely to be satisfied in empirical economic applications where the countries or regions depend on each other. Thus in this section we therefore generalize our earlier results to suit the case of dependence among the cross-sectional units, by filtering out common factor in the panel structure. Basically, the estimation procedure is based on the Stock and Watson (2002), Bai (2003) and Bai and Ng (2004) approaches. It consists of two steps. In the first step, the common factors are estimated using the Principal Component (PC) method. In order to estimate the true number of common factors accurately in this first step, we compute the number of factors using Bai and Ng's (2002) selection criteria. In the second step, defactored data is constructed.

Specifically, let, as before, $y_{i,t}$ be the observation on the $i$-th cross section unit at time t for $i = 1,\ldots,N$, $t = 1,\ldots,T$ and suppose that it is generated according to the following linear heterogeneous panel data model:

$$y_{i,t} = d_{i,t} + \mu_{i,t} + F_t\lambda_i + \varepsilon_{i,t} \tag{28}$$

As in the previous section, three cases are considered:

- Case 1: I(0) $\to$ I(1)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t > [\tau T])\eta_{i,t} \tag{29}$$

- Case 2: I(1) → I(0)

$$\mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau T])\eta_{i,t} \tag{30}$$

- Case 3: unknown direction I(0) → I(1) or I(1) → I(0)

where, as before, $\varepsilon_{i,t}$ is a stationary process, $\tau \in (0, 1)$ and the deterministic component $d_{i,t}$ is taken to be the unity vector. Further we assume that $F_t$ in (28) is a stationary $(r \times 1)$ vector of common factors and $\lambda_i$ is the corresponding vector of factor loadings.
Let us write the factors $F_t$ and factor loadings $\lambda_i$ in (28) in matrix notation. We write $F = (F_1, ..., F_T)'$ and $\Lambda = (\lambda_1, ..., \lambda_N)'$. The following assumptions are required:

**Assumption 3** *the loadings $\lambda_i$ is either deterministic such that $|\lambda_i\|^4 \leq M$ or stochastic such that $E\|\lambda_i\|^4 \leq M$, in either case $\Lambda_i'\Lambda_i/N \to \Sigma_\Lambda$, as $N \to \infty$ for some $(r \times r)$ positive definite matrix $\Sigma_\lambda$.*

**Assumption 4** *$E\|F_t\|^4 \leq M$ and $\frac{1}{T}\sum_{i=1}^{T} F_t F_t' \to \Sigma_F$, for a $(r \times r)$ positive definite matrix $\Sigma_F$.*

**Assumption 5** *The errors $\{\varepsilon_{i,t}\}$ and $\{\eta_{i,t}\}$, the factor $\{F_t\}$ and the loadings $\{\lambda_i\}$ are four mutually independent stochastic variables.*

Assumption 3 and 4 imply $r$ common factors and assumption 5 is standard in factor analysis. The common factors are estimated as they are in Stock and Watson (2002), i.e. using principal components. Specifically, the principal component of $F$, denoted as $\tilde{F}$, is $\sqrt{T}$ times the first $r$ eigenvectors, corresponding to the first $r$ largest eigenvalues of the $(T \times T)$ matrix of demeaned and stardardized $\tilde{y}_i\tilde{y}_i{}'$. Under the normalization $\tilde{F}\tilde{F}'/T = I_r$, the estimated loading matrix is $\tilde{\Lambda} = \tilde{F}'\tilde{y}_i/T$. Thus the estimated residuals are defined as

$$\tilde{z}_{i,t} = \tilde{y}_{i,t} - \tilde{F}_t\tilde{\lambda}_i \tag{31}$$

From the data generating process (28)-(29) and (31), one can, for example, see that for each cross section $i$, the process $\tilde{z}_{i,t}$ is stationary up to and including time $[\tau T]$ but is I(1) after the break if, and only if, $\sigma^2_{\eta i} > 0$. Naturally the converse it's true if we adopt (30) instead of (29).
Thus our strategy is to apply the panel test statistics presented in section 2 to the de-factored data $\tilde{z}_{i,t}$.

# 4 Monte Carlo simulation results

In this section we use Monte Carlo experiments to examine the finite sample properties of the panel persistence tests. We consider two sets of Monte Carlo experiments. The first set focuses on the model (1)-(3), i.e where we assume cross-section independence, while the second set of experiments is based on the model (28)-(31) where we allow for the presence of dependence across the different units in the panel. We start the analysis considering the empirical rejection frequencies of the tests when the data are generated according to the I(0)-I(1) switch data generating process embraced in (1)-(3) under the hypothesis of cross-section independence. We investigate the impact of varying the signal-noise-ratio among $\sigma_{\eta i} = 0, 0.10, 0.50$ and $\sigma_{\varepsilon i} \sim U[0.5, 1.5]$ and the breakpoint among $\tau = 0.3, 0.5, 0.7$. The simulation results were performed using 1000 Monte Carlo replications and the RNDN function of Gauss 6.0. As is often the case in the literature, we fix for all the tests $\bigwedge = [0.2, 0.8]$ and $T = 50, 100$ and $N = 25, 50, 100$. In Table 1 we present the moments of Kim's (2000, 2002) and Busetti and Taylor's (2004) tests which were used to standardize the panel tests. Their values were computed using 50,000 replications.

Table 1 about here

The size results for the benchmark model (1)-(3) are reported in Table 2.

Table 2 about here

All the panel test statistics seem to have good size for both small and large $T, N$.

Looking at power of the tests for the case $I(0) \rightarrow I(1)$ (Tables 3-5), many interesting results emerge.

Table 3 about here
Table 4 about here
Table 5 about here

Comparing the power of panel tests derived using Kim's (2000), Busetti and Taylor's (2004) and Harvey *et al.*'s (2006) methods we find no significant differences. As expected, the power of tests grows, with the exception of reverse panel tests, as the signal to noise ratio rises and $\tau$ is smaller. This occurs because the higher $\sigma_\eta$ is, the stronger is the random walk component. We have that the smaller $\tau$ is, the greater is the proportion of the sample containing a random walk component. Finally, the previous finding are reversed for the panel reverse tests. This because the fact that we are testing a change from I(0) to I(1). The opposite results are found for a shift from $I(1) \rightarrow I(0)$. Here, as expected, the power of tests grows largely for the reverse tests and it is striking that the results mimic those in Tables 3-5. Thus $H_1(\mathcal{K}^*)$, $H_2(\mathcal{K}^*)$ and $H_3(\mathcal{K}^*)$ show better properties than $H_1(\mathcal{K})$, $H_2(\mathcal{K})$ $H_3(\mathcal{K})$ and $\mathcal{M}^{1*}$, $\mathcal{M}^{2*}$ and $\mathcal{M}^{3*}$ (see Tables 6-8).

Table 6 about here
Table 7 about here
Table 8 about here

We now present the empirical size of tests when cross-section dependence is included in the model, as in equations (28)-(30). We consider two cases of cross section dependence, a "low cross section dependence", where $\lambda_i \sim iidU[0.0, 0.2]$, and "high cross section dependence", with $\lambda_i \sim iidU[-1, 3]$. In the simulation we include only one factor $F_t \sim N(0, 1)$. Results are reported in panels A (low case) and B (high case) of Table 9.

Table 9 about here

As expected, the extent of over-rejection of the tests very much depends on the degree of cross-section dependence. Both for low as well as for strong cross-section dependence the panel tests are distorted with over-rejection, which grows as the degree of cross-section dependence rise. Thus panel tests that do not allow for cross-section dependence can be seriously biased if the degree cross-section dependence is large. To take into account of cross-section dependence we use the method proposed in Stock and Watson (2002) and Bai and Ng (2004). The method basically consists of filtering out the individual-specific cross-sections $y_{it}$ by the factor component computed using the principal component method. The number of factors are computed using the methodology proposed in Bai and Ng (2002). To be precise throughout the Monte Carlo simulation analyses the number of factors are computed using the $IC(3)$ criterion proposed in Bai and Ng (2002) with a maximum number of three factors. As before, we use only one factor $F_t \sim N(0, 1)$, and $\lambda \sim U[0, 1]$.
In Tables 10 and in 11-13, we present, respectively, the size and power of de-factored panel tests using the Stock and Watson (2002) methodology. Looking at the results we note first that the tests have now generally good size. As expected, the power of tests grows for larger values of $T$ and $N$.

Table 10 about here
Table 11 about here
Table 12 about here
Table 13 about here

Finally, results of the power of tests for the case of a change from I(1) to I(0) are reported in Tables 14-16. As expected, the power of tests grows large for the reverse tests.

Table 14 about here
Table 15 about here
Table 16 about here

# 5 Empirical applications

We apply the panel tests described in this paper to a panel of 21 OECD quarterly inflation rate series observed for the period 1970.1-2007.3. [1] The series are calculated as the first difference of the logarithm of the (seasonally adjusted) consumer price index. The data are taken from OECD Main Economic Indicators. In Table 17 the panel tests results are reported.

Table 17 about here

Looking at their values, we note first that for first set of test statistics, i.e. test statistics which are computed not taking into account possible cross-section

[1] The countries included in the panel are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, the United States.

dependence, reverse tests strongly suggest a change of persistence from I(1) to I(0). Change in persistence are also shown by the $\mathcal{M}^j$ tests as well as their modified version. Mixed responses are obtained from the $\mathrm{H}_1(\mathcal{K})$ test, when this is compared with $\mathrm{H}_2(\mathcal{K})$ and $\mathrm{H}_3(\mathcal{K})$ test statistics. While the former test rejectes the null hypothesis, the latter tests do not reject the null hypothesis for a process which is I(0) throughout. In order to take into account possible cross-dependence across the countries, we first compute the number of factors. The $IC(3)$ criterion suggests three factors (we allow for a maximum number of five factors). We use the estimated factors and factor loadings to compute $\tilde{\hat{z}}_{i,t}$, as in equation (31). Looking at the results of panel test statistics we note that the previous results are now partially reversed. Here both $\mathrm{H}_1(\mathcal{K})$, and $\mathrm{H}_2(\mathcal{K})$, $\mathrm{H}_3(\mathcal{K})$ test statistics do not reject the null hypothesis, allowing for a process which is I(0) throughout the sample. Reversed test statistics indicate a change from I(1) to I(0) and $\mathcal{M}^j$ strongly reject the null hypothesis of constant persistence (constant I(0)). Given these results, we conclude that the inflation process is characterized by a change in persistence from I(1) to I(0). Using the cross-sectional dependence adjusted series, the change in persistence, computed using (25), took place in 1980.4.

# 6 Conclusions

In this paper we present new panel test statistics for a change in persistence which are based on a modified time series version of the ratio-based statistics presented in Busetti and Taylor (2004) and Harvey *et al.* (2006). These statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1), or viceversa. The alternative of an unknown direction is also considered. Asymptotic distributions of the new panel tests under the hypothesis of cross-section independence are derived and Monte Carlo analysis suggests that these tests perform very well. Cross-section dependence is also considered.

We show first that when testing for a change in persistence from I(0) (I(1)) to I(1) (I(0)) panel tests have good properties. Secondly, we report the importance of taking into account possible cross-sectional dependence when computing the panel test statistics, especially for highly dependent panels. Finally, we apply the panel tests to a panel of 21 OECD inflation rates observed during the period 1970.1 - 2007.3. The results were consistent with a change of persistence from I(1) to I(0) in April 1980.

## Appendix A. Proof of theorems

**Proof of Theorem 2.1.** Fixed $i = 1, \dots, N$, $t_1 = 1, \dots, [T\tau]$ and $t_2 = [T\tau]+1, \dots, T$, it results $S^{(1)}_{i,t_1}$ and $S^{(0)}_{i,t_2}$ mutually independent. Therefore, (8) and (9) can be rewritten as

$$\mu = \mathbf{E}\Big[\frac{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}\Big]$$

and

$$\sigma = \sqrt{\mathbf{V}\Big[\frac{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}\Big]}.$$

By Theorem 3.1 in Kim (2000), it results

$$\lim_{T\to+\infty} \frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \frac{\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2} = \frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2 \mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r},$$

where $\{V_i\}_{i=1}^{+\infty}$ is a sequence of standard brownian bridges that are independent and identically distributed. Furthermore, by the hypotheses stated in Assumption 1, we have that

$$\lim_{T\to+\infty} \mathbf{E}\Big[\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \frac{\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}\Big] = \bar{\mu},$$

$$\lim_{T\to+\infty} \mathbf{V}\Big[\frac{(T-[T\tau])^{-2}}{[T\tau]^{-2}} \frac{\sum_{t=[T\tau]+1}^{T} S^{(1)}_{i,t}(\tau)^2}{\sum_{t=1}^{[T\tau]} S^{(0)}_{i,t}(\tau)^2}\Big] = \bar{\sigma}^2,$$

with

$$\bar{\mu} = \mathbf{E}\Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2 \mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r}\Big] \tag{32}$$

and

$$\bar{\sigma}^2 = \mathbf{V}\Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2 \mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r}\Big] \tag{33}$$

for each $i = 1, \dots, N$. Therefore

$$\lim_{N\to+\infty}\lim_{T\to+\infty} \mathcal{K}_{T,N}(\tau) = \lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2 \mathrm{d}r}{\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r} - \bar{\mu}\Big],$$

for $i = 1, \dots, N$. The Central Limit Theorem guarantees that

$$\lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \Big[\frac{(1-\tau)^{-2}\int_\tau^1 V_i(r-\tau)^2 \mathrm{d}r}{N\tau^{-2}\int_0^\tau V_i(r)^2\mathrm{d}r} - \bar{\mu}\Big] \sim N(0,1),$$

and the Theorem is completely proved. ■

**Proof of Theorem 2.2.**

(i) The result follows from the continuous mapping theorem and the continuity of the functionals.

(ii) Since $\mathcal{K}(\tau) \sim N(0,1)$, for each $\tau$, then $\mathcal{K}(\tau)$ is an iid continuous-time stochastic process. Therefore, we can define the random variable $\mathcal{K} \sim N(0,1)$ such that $\mathcal{K}(\tau) \equiv \mathcal{K}$, for each $\tau \in (0,1)$.
Then we have

$$\mathrm{H}_1(\mathcal{K}(\tau)) = \sup_{\tau\in(0,1)} (\mathcal{K}) = \mathcal{K} \sim N(0,1);$$

$$\mathrm{H}_2(\mathcal{K}(\tau)) = \int_{\tau\in(0,1)} \mathcal{K}\mathrm{d}\tau = \mathcal{K} \cdot \int_{\tau\in(0,1)} \mathrm{d}\tau = \mathcal{K} \sim N(0,1);$$

$$\mathrm{H}_3(\mathcal{K}(\tau)) = \log\Big\{ \int_{\tau\in(0,1)} \exp[\mathcal{K}]\mathrm{d}\tau \Big\} = \log\{\exp[\mathcal{K}]\} \cdot \int_{\tau\in(0,1)} \mathrm{d}\tau = \mathcal{K} \sim N(0,1)$$

The result is completely proved. ■

**Proof of Theorem 2.3.** By Theorem 3.1 in Busetti and Taylor (2004), it results

$$\lim_{T\to+\infty} \frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2} = \frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2\mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2\mathrm{d}r},$$

where

$$V_i^{**}(r) = V_i(r) - V_i(\tau) - (r-\tau)(1-\tau)^{-1}(V_i(1) - V_i(\tau))$$
$$V_i^{***}(r) = V_i(r) - r\tau^{-1}V_i(\tau)$$

and

$$V_i(r) = W_0(r) + c\left\{ \int_0^{min\{r,\tau\}} W_c(s)\mathrm{d}s + 1(r>\tau)[(r-\tau)W_c(\tau)] \right\},$$

where $W$ is a standard Wiener process. Hypotheses stated in Assumption 1 imply

$$\lim_{T\to+\infty} \mathbf{E}\Big[\frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}\Big] = \bar{\mu},$$

$$\lim_{T\to+\infty} \mathbf{V}\Big[\frac{[T\tau]^{-2}\sum_{t=1}^{[T\tau]} S_{i,t}^{(0)}(\tau)^2}{(T-[T\tau])^{-2}\sum_{t=[T\tau]+1}^{T} S_{i,t}^{(1)}(\tau)^2}\Big] = \bar{\sigma}^2,$$

with

$$\bar{\mu} = \mathbf{E}\Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2\mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2\mathrm{d}r}\Big] \tag{34}$$

and

$$\bar{\sigma}^2 = \mathbf{V}\Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2\mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2\mathrm{d}r}\Big] \tag{35}$$

for each $i = 1, \ldots, N$. Hence

$$\lim_{N\to+\infty}\lim_{T\to+\infty} \mathcal{K}_{T,N}^{\star}(\tau) = \lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \Big[\frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2\mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2\mathrm{d}r} - \bar{\mu}\Big],$$

for $i = 1, \ldots, N$. The Central Limit Theorem guarantees that

$$\lim_{N\to+\infty} \frac{\sqrt{N}}{\bar{\sigma}} \cdot \frac{1}{N} \cdot \sum_{i=1}^{N} \Big[ \frac{\tau^{-2}\int_0^\tau [V_i^{***}(r)]^2 \mathrm{d}r}{(1-\tau)^{-2}\int_\tau^1 [V_i^{**}(r)]^2 \mathrm{d}r} - \bar{\mu} \Big] \sim N(0,1),$$

and the Theorem is completely proved. ■

**Proof of Theorem 2.4.** Analogous to the proof of Theorem 2.2. ■

**Proof of Theorem 2.5.** It is a direct consequence of the Central Limit Theorem. ■

**Proof of Theorem 2.6.** Since

$$M_T^{(i)}(\tau) := \frac{\sum_{t=[T\tau]+1}^{T} \tilde{\mu}_{i,t}^2/(T-[T\tau])^2}{\sum_{t=1}^{[T\tau]} \tilde{\mu}_{i,t}^2/[\tau T]} \geq 0, \qquad \forall\, i = 1, \ldots, N,$$

then

$$\hat{\tau} = \mathrm{argmax}_{\tau\in(0,1)} M_T^{(i)}(\tau) \Rightarrow$$

$$\Rightarrow \hat{\tau} = \mathrm{argmax}_{\tau\in(0,1)} \Big\{ \frac{1}{N} \cdot \sum_{i=1}^{N} M_T^{(i)}(\tau) \Big\} = \mathrm{argmax}_{\tau\in(0,1)} \Pi_{N,T}(\tau).$$

Therefore, Theorem 3.5 in Kim (2000) guarantees the thesis. ■

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Table 1: Simulated moments for individual Kim(2000, 2002) and Busetti and Taylor tests (2004)

| T | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|
| Mean - drift case | | | | | | | | | |
| 50 | 1.817 | 1.608 | 6.173 | 1.836 | 1.624 | 6.204 | 2.783 | 2.632 | 9.214 |
| 100 | 1.803 | 1.566 | 6.386 | 1.803 | 1.564 | 6.391 | 2.749 | 2.545 | 9.436 |
| 500 | 1.800 | 1.537 | 6.788 | 1.779 | 1.506 | 6.660 | 2.741 | 2.478 | 9.868 |
| Mean - linear trend case | | | | | | | | | |
| 50 | 1.415 | 0.906 | 3.785 | 1.412 | 0.908 | 3.796 | 1.992 | 1.334 | 5.297 |
| 100 | 1.377 | 0.844 | 3.738 | 1.374 | 0.843 | 3.831 | 1.924 | 1.222 | 5.145 |
| 500 | 1.362 | 0.815 | 3.718 | 1.353 | 0.822 | 3.839 | 1.886 | 1.177 | 5.115 |
| Std. deviation - drift case | | | | | | | | | |
| 50 | 1.580 | 2.282 | 5.856 | 1.586 | 2.370 | 6.003 | 1.737 | 2.903 | 6.874 |
| 100 | 1.548 | 2.178 | 5.841 | 1.526 | 2.106 | 5.728 | 1.669 | 2.644 | 6.590 |
| 500 | 1.528 | 2.170 | 6.041 | 1.503 | 1.980 | 5.701 | 1.667 | 2.562 | 6.318 |
| Std. deviation - linear trend case | | | | | | | | | |
| 50 | 0.869 | 0.863 | 2.759 | 0.866 | 0.861 | 2.777 | 0.869 | 1.030 | 3.068 |
| 100 | 0.814 | 0.687 | 2.451 | 0.798 | 0.685 | 2.447 | 0.794 | 0.781 | 2.618 |
| 500 | 0.764 | 0.607 | 2.428 | 0.734 | 0.651 | 2.401 | 0.709 | 0.701 | 2.604 |

Table 2: Size of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. Intercept case.

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.075 | 0.074 | 0.078 | 0.069 | 0.061 | 0.058 | 0.056 | 0.061 | 0.063 |
| | 50 | 0.078 | 0.074 | 0.068 | 0.064 | 0.060 | 0.047 | 0.074 | 0.069 | 0.067 |
| | 100 | 0.073 | 0.072 | 0.078 | 0.065 | 0.063 | 0.056 | 0.065 | 0.062 | 0.077 |
| 100 | 25 | 0.054 | 0.054 | 0.051 | 0.076 | 0.072 | 0.073 | 0.069 | 0.072 | 0.070 |
| | 50 | 0.041 | 0.040 | 0.046 | 0.056 | 0.069 | 0.069 | 0.058 | 0.059 | 0.064 |
| | 100 | 0.051 | 0.060 | 0.064 | 0.053 | 0.065 | 0.055 | 0.049 | 0.051 | 0.051 |
| T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
| 50 | 25 | 0.075 | 0.075 | 0.078 | 0.069 | 0.063 | 0.062 | 0.062 | 0.065 | 0.064 |
| | 50 | 0.078 | 0.078 | 0.069 | 0.065 | 0.061 | 0.047 | 0.075 | 0.070 | 0.067 |
| | 100 | 0.073 | 0.072 | 0.070 | 0.065 | 0.066 | 0.056 | 0.066 | 0.064 | 0.073 |
| 100 | 25 | 0.055 | 0.054 | 0.052 | 0.078 | 0.078 | 0.076 | 0.070 | 0.075 | 0.072 |
| | 50 | 0.043 | 0.043 | 0.048 | 0.056 | 0.071 | 0.069 | 0.058 | 0.060 | 0.065 |
| | 100 | 0.051 | 0.060 | 0.066 | 0.054 | 0.065 | 0.056 | 0.050 | 0.052 | 0.052 |
| T | N | $MH_{1m}(\mathcal{K})$ | $MH_{2m}(\mathcal{K})$ | $MH_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $M\mathcal{M}_m^{1*}$ | $M\mathcal{M}_m^{2*}$ | $M\mathcal{M}_m^{3*}$ |
| 50 | 25 | 0.075 | 0.075 | 0.076 | 0.069 | 0.063 | 0.062 | 0.059 | 0.064 | 0.063 |
| | 50 | 0.068 | 0.068 | 0.069 | 0.065 | 0.061 | 0.047 | 0.075 | 0.070 | 0.067 |
| | 100 | 0.073 | 0.072 | 0.070 | 0.065 | 0.065 | 0.056 | 0.065 | 0.064 | 0.072 |
| 100 | 25 | 0.055 | 0.054 | 0.051 | 0.077 | 0.073 | 0.072 | 0.060 | 0.074 | 0.071 |
| | 50 | 0.043 | 0.044 | 0.046 | 0.056 | 0.068 | 0.068 | 0.058 | 0.059 | 0.065 |
| | 100 | 0.051 | 0.060 | 0.064 | 0.054 | 0.065 | 0.056 | 0.049 | 0.051 | 0.052 |

**Notes :** Empirical sizes corresponding to a 5% nominal size.

Table 3: Power of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. I(0) → I(1). Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.582 | 0.610 | 0.611 | 0.193 | 0.209 | 0.107 | 0.544 | 0.574 | 0.561 |
| | | | 50 | 0.845 | 0.858 | 0.854 | 0.272 | 0.316 | 0.130 | 0.798 | 0.821 | 0.802 |
| | | | 100 | 0.979 | 0.985 | 0.987 | 0.417 | 0.490 | 0.147 | 0.971 | 0.981 | 0.968 |
| | | 100 | 25 | 0.998 | 0.998 | 0.997 | 0.801 | 0.832 | 0.470 | 0.999 | 0.999 | 0.998 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.950 | 0.965 | 0.640 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.997 | 0.997 | 0.806 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.534 | 0.557 | 0.640 | 0.035 | 0.048 | 0.012 | 0.346 | 0.372 | 0.444 |
| | | | 50 | 0.819 | 0.818 | 0.902 | 0.030 | 0.042 | 0.007 | 0.588 | 0.609 | 0.669 |
| | | | 100 | 0.943 | 0.959 | 0.983 | 0.014 | 0.022 | 0.000 | 0.775 | 0.813 | 0.879 |
| | | 100 | 25 | 0.994 | 0.994 | 0.999 | 0.125 | 0.152 | 0.004 | 0.983 | 0.990 | 0.992 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.117 | 0.158 | 0.001 | 0.998 | 0.998 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.097 | 0.164 | 0.000 | 0.998 | 0.998 | 1.000 |
| 0.7 | | 50 | 25 | 0.280 | 0.305 | 0.406 | 0.025 | 0.032 | 0.020 | 0.150 | 0.165 | 0.226 |
| | | | 50 | 0.438 | 0.456 | 0.628 | 0.016 | 0.023 | 0.005 | 0.199 | 0.210 | 0.361 |
| | | | 100 | 0.639 | 0.662 | 0.866 | 0.008 | 0.011 | 0.003 | 0.317 | 0.349 | 0.554 |
| | | 100 | 25 | 0.909 | 0.920 | 0.973 | 0.003 | 0.009 | 0.002 | 0.701 | 0.749 | 0.885 |
| | | | 50 | 0.989 | 0.990 | 0.999 | 0.001 | 0.003 | 0.000 | 0.888 | 0.921 | 0.984 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.994 | 0.995 | 0.930 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 10 | 1.000 | 1.000 | 1.000 | 0.974 | 0.985 | 0.861 | 1.000 | 1.000 | 1.000 |
| | | | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.636 | 0.685 | 0.051 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.834 | 0.872 | 0.034 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.952 | 0.979 | 0.018 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.914 | 0.933 | 0.232 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.982 | 0.987 | 0.263 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.244 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.002 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |

Table 4: Power of Modified Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. I(0) $\rightarrow$ I(1). Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.588 | 0.612 | 0.615 | 0.197 | 0.214 | 0.108 | 0.557 | 0.578 | 0.574 |
| | | | 50 | 0.851 | 0.863 | 0.854 | 0.275 | 0.317 | 0.132 | 0.802 | 0.827 | 0.808 |
| | | | 100 | 0.980 | 0.985 | 0.988 | 0.420 | 0.491 | 0.148 | 0.972 | 0.981 | 0.969 |
| | | 100 | 25 | 0.998 | 0.998 | 0.997 | 0.803 | 0.836 | 0.471 | 0.999 | 0.999 | 0.998 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.951 | 0.965 | 0.643 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.997 | 0.997 | 0.809 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.542 | 0.560 | 0.647 | 0.038 | 0.048 | 0.013 | 0.356 | 0.382 | 0.451 |
| | | | 50 | 0.823 | 0.822 | 0.903 | 0.033 | 0.042 | 0.007 | 0.591 | 0.619 | 0.676 |
| | | | 100 | 0.945 | 0.960 | 0.983 | 0.015 | 0.022 | 0.000 | 0.778 | 0.814 | 0.881 |
| | | 100 | 25 | 0.994 | 0.994 | 0.999 | 0.126 | 0.155 | 0.004 | 0.984 | 0.990 | 0.992 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.118 | 0.159 | 0.001 | 0.999 | 0.999 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.099 | 0.164 | 0.000 | 0.982 | 0.987 | 0.999 |
| 0.7 | | 50 | 25 | 0.289 | 0.309 | 0.410 | 0.025 | 0.033 | 0.021 | 0.159 | 0.170 | 0.230 |
| | | | 50 | 0.442 | 0.458 | 0.634 | 0.017 | 0.023 | 0.005 | 0.201 | 0.221 | 0.364 |
| | | | 100 | 0.640 | 0.664 | 0.866 | 0.008 | 0.012 | 0.003 | 0.321 | 0.353 | 0.560 |
| | | 100 | 25 | 0.912 | 0.920 | 0.973 | 0.003 | 0.009 | 0.002 | 0.708 | 0.751 | 0.889 |
| | | | 50 | 0.989 | 0.990 | 0.999 | 0.001 | 0.003 | 0.000 | 0.889 | 0.924 | 0.984 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.982 | 0.987 | 0.999 |
| 0.3 | 0.50 | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.994 | 0.995 | 0.933 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.651 | 0.695 | 0.052 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.835 | 0.876 | 0.034 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.953 | 0.979 | 0.018 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.916 | 0.935 | 0.237 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.982 | 0.987 | 0.265 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.247 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 10 | 0.999 | 1.000 | 1.000 | 0.002 | 0.005 | 0.001 | 0.990 | 0.995 | 1.000 |
| | | | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.001 | 0.001 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |

Table 5: Power of Modified min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. I(0) $\rightarrow$ I(1). Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_{1\text{m}}(\mathcal{K})$ | $\text{MH}_{2\text{m}}(\mathcal{K})$ | $\text{MH}_{3\text{m}}(\mathcal{K})$ | $\text{MH}_{1\text{m}}(\mathcal{K}^*)$ | $\text{MH}_{2\text{m}}(\mathcal{K}^*)$ | $\text{MH}_{3\text{m}}(\mathcal{K}^*)$ | $\text{M}\mathcal{M}_m^{1*}$ | $\text{M}\mathcal{M}_m^{2*}$ | $\text{M}\mathcal{M}_m^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.586 | 0.612 | 0.612 | 0.197 | 0.213 | 0.108 | 0.552 | 0.576 | 0.574 |
| | | | 50 | 0.849 | 0.859 | 0.854 | 0.273 | 0.316 | 0.132 | 0.801 | 0.824 | 0.806 |
| | | | 100 | 0.979 | 0.985 | 0.988 | 0.419 | 0.491 | 0.147 | 0.972 | 0.981 | 0.969 |
| | | 100 | 25 | 0.998 | 0.998 | 0.997 | 0.802 | 0.832 | 0.470 | 0.999 | 0.999 | 0.998 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.951 | 0.965 | 0.642 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.997 | 0.997 | 0.806 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.539 | 0.558 | 0.647 | 0.037 | 0.048 | 0.013 | 0.354 | 0.379 | 0.450 |
| | | | 50 | 0.821 | 0.819 | 0.903 | 0.032 | 0.042 | 0.007 | 0.590 | 0.617 | 0.673 |
| | | | 100 | 0.998 | 0.998 | 0.997 | 0.802 | 0.832 | 0.470 | 0.999 | 0.999 | 0.998 |
| | | 100 | 25 | 0.994 | 0.994 | 0.999 | 0.125 | 0.152 | 0.004 | 0.984 | 0.990 | 0.992 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.117 | 0.158 | 0.001 | 0.998 | 0.998 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.098 | 0.164 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.287 | 0.307 | 0.409 | 0.025 | 0.032 | 0.021 | 0.154 | 0.168 | 0.230 |
| | | | 50 | 0.441 | 0.456 | 0.633 | 0.017 | 0.023 | 0.005 | 0.200 | 0.220 | 0.363 |
| | | | 100 | 0.640 | 0.663 | 0.866 | 0.008 | 0.012 | 0.003 | 0.320 | 0.353 | 0.560 |
| | | 100 | 25 | 0.911 | 0.920 | 0.973 | 0.003 | 0.009 | 0.001 | 0.702 | 0.751 | 0.887 |
| | | | 50 | 0.989 | 0.990 | 0.999 | 0.001 | 0.003 | 0.000 | 0.888 | 0.922 | 0.984 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.098 | 0.164 | 0.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.994 | 0.995 | 0.930 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.996 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.995 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.642 | 0.687 | 0.051 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.834 | 0.872 | 0.034 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.952 | 0.979 | 0.018 | 0.932 | 0.949 | 0.933 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.915 | 0.933 | 0.233 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.982 | 0.987 | 0.263 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.244 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.000 | 0.001 | 0.001 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 |

Table 6: Power of Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \rightarrow I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.030 | 0.037 | 0.021 | 0.266 | 0.284 | 0.391 | 0.160 | 0.182 | 0.249 |
| | | | 50 | 0.021 | 0.036 | 0.012 | 0.427 | 0.428 | 0.593 | 0.206 | 0.218 | 0.355 |
| | | | 100 | 0.008 | 0.011 | 0.005 | 0.639 | 0.674 | 0.839 | 0.319 | 0.358 | 0.549 |
| | | 100 | 25 | 0.002 | 0.003 | 0.001 | 0.891 | 0.913 | 0.968 | 0.682 | 0.736 | 0.875 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 0.990 | 0.993 | 0.998 | 0.893 | 0.931 | 0.976 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.983 | 0.990 | 1.000 |
| 0.5 | | 50 | 25 | 0.041 | 0.049 | 0.017 | 0.550 | 0.570 | 0.657 | 0.382 | 0.405 | 0.453 |
| | | | 50 | 0.032 | 0.047 | 0.007 | 0.792 | 0.810 | 0.890 | 0.563 | 0.598 | 0.703 |
| | | | 100 | 0.020 | 0.034 | 0.001 | 0.964 | 0.973 | 0.993 | 0.820 | 0.856 | 0.907 |
| | | 100 | 25 | 0.095 | 0.127 | 0.004 | 0.995 | 0.996 | 0.998 | 0.975 | 0.981 | 0.985 |
| | | | 50 | 0.095 | 0.134 | 0.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 1.000 |
| | | | 100 | 0.089 | 0.141 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.220 | 0.247 | 0.120 | 0.623 | 0.639 | 0.636 | 0.593 | 0.619 | 0.599 |
| | | | 50 | 0.326 | 0.370 | 0.134 | 0.858 | 0.860 | 0.860 | 0.814 | 0.841 | 0.819 |
| | | | 100 | 0.437 | 0.512 | 0.174 | 0.984 | 0.985 | 0.981 | 0.967 | 0.978 | 0.975 |
| | | 100 | 25 | 0.789 | 0.817 | 0.429 | 0.996 | 0.999 | 0.996 | 0.995 | 0.997 | 0.996 |
| | | | 50 | 0.941 | 0.950 | 0.595 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.998 | 0.758 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 25 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.002 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.000 | 0.001 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.659 | 0.708 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.846 | 0.887 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.966 | 0.981 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.910 | 0.921 | 0.195 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.095 | 0.133 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.999 | 0.238 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.999 | 0.999 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.966 | 0.981 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table 7: Power of Modified Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \rightarrow I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $MH_1^d(\mathcal{K})$ | $MH_2^d(\mathcal{K})$ | $MH_3^d(\mathcal{K})$ | $MH_1^d(\mathcal{K}^*)$ | $MH_2^d(\mathcal{K}^*)$ | $MH_3^d(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.030 | 0.037 | 0.021 | 0.266 | 0.284 | 0.391 | 0.160 | 0.182 | 0.249 |
| | | | 50 | 0.021 | 0.036 | 0.012 | 0.427 | 0.428 | 0.593 | 0.206 | 0.218 | 0.355 |
| | | | 100 | 0.008 | 0.011 | 0.005 | 0.639 | 0.674 | 0.839 | 0.319 | 0.358 | 0.549 |
| | | 100 | 25 | 0.002 | 0.003 | 0.001 | 0.891 | 0.913 | 0.968 | 0.682 | 0.736 | 0.875 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 0.990 | 0.993 | 0.998 | 0.893 | 0.931 | 0.976 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.983 | 0.990 | 1.000 |
| 0.5 | | 50 | 25 | 0.041 | 0.049 | 0.017 | 0.550 | 0.570 | 0.657 | 0.382 | 0.405 | 0.453 |
| | | | 50 | 0.032 | 0.047 | 0.007 | 0.792 | 0.810 | 0.890 | 0.563 | 0.598 | 0.703 |
| | | | 100 | 0.020 | 0.034 | 0.001 | 0.964 | 0.973 | 0.993 | 0.820 | 0.856 | 0.907 |
| | | 100 | 25 | 0.095 | 0.127 | 0.004 | 0.995 | 0.996 | 0.998 | 0.975 | 0.981 | 0.985 |
| | | | 50 | 0.095 | 0.134 | 0.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 1.000 |
| | | | 100 | 0.089 | 0.141 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.220 | 0.247 | 0.120 | 0.623 | 0.639 | 0.636 | 0.593 | 0.619 | 0.599 |
| | | | 50 | 0.326 | 0.370 | 0.134 | 0.858 | 0.860 | 0.860 | 0.814 | 0.841 | 0.819 |
| | | | 100 | 0.437 | 0.512 | 0.174 | 0.984 | 0.985 | 0.981 | 0.967 | 0.978 | 0.975 |
| | | 100 | 25 | 0.789 | 0.817 | 0.429 | 0.996 | 0.999 | 0.996 | 0.995 | 0.997 | 0.996 |
| | | | 50 | 0.941 | 0.950 | 0.595 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.998 | 0.758 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 25 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.002 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.000 | 0.001 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.659 | 0.708 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.846 | 0.887 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.966 | 0.981 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.910 | 0.921 | 0.195 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.095 | 0.133 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.999 | 0.238 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.999 | 0.999 | 0.937 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.966 | 0.981 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table 8: Power of Modified min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. $I(1) \to I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_{1m}(\mathcal{K})$ | $\text{MH}_{2m}(\mathcal{K})$ | $\text{MH}_{3m}(\mathcal{K})$ | $\text{MH}_{1m}(\mathcal{K}^*)$ | $\text{MH}_{2m}(\mathcal{K}^*)$ | $\text{MH}_{3m}(\mathcal{K}^*)$ | $\text{M}\mathcal{M}_m^{1*}$ | $\text{M}\mathcal{M}_m^{2*}$ | $\text{M}\mathcal{M}_m^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.030 | 0.037 | 0.021 | 0.266 | 0.284 | 0.391 | 0.160 | 0.182 | 0.249 |
| | | | 50 | 0.021 | 0.036 | 0.012 | 0.427 | 0.428 | 0.593 | 0.206 | 0.218 | 0.355 |
| | | | 100 | 0.008 | 0.011 | 0.005 | 0.639 | 0.674 | 0.839 | 0.319 | 0.358 | 0.549 |
| | | 100 | 25 | 0.002 | 0.003 | 0.001 | 0.891 | 0.913 | 0.968 | 0.682 | 0.736 | 0.875 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 0.990 | 0.993 | 0.998 | 0.893 | 0.931 | 0.976 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.983 | 0.990 | 1.000 |
| 0.5 | | 50 | 25 | 0.041 | 0.049 | 0.017 | 0.550 | 0.570 | 0.657 | 0.382 | 0.405 | 0.453 |
| | | | 50 | 0.032 | 0.047 | 0.007 | 0.792 | 0.810 | 0.890 | 0.563 | 0.598 | 0.703 |
| | | | 100 | 0.020 | 0.034 | 0.001 | 0.964 | 0.973 | 0.993 | 0.820 | 0.856 | 0.907 |
| | | 100 | 25 | 0.095 | 0.127 | 0.004 | 0.995 | 0.996 | 0.998 | 0.975 | 0.981 | 0.986 |
| | | | 50 | 0.095 | 0.134 | 0.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.999 | 1.000 |
| | | | 100 | 0.089 | 0.141 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.220 | 0.247 | 0.120 | 0.623 | 0.639 | 0.636 | 0.593 | 0.619 | 0.599 |
| | | | 50 | 0.326 | 0.370 | 0.134 | 0.858 | 0.860 | 0.860 | 0.814 | 0.841 | 0.819 |
| | | | 100 | 0.437 | 0.512 | 0.174 | 0.984 | 0.985 | 0.981 | 0.967 | 0.978 | 0.975 |
| | | 100 | 25 | 0.789 | 0.817 | 0.429 | 0.996 | 0.999 | 0.996 | 0.995 | 0.997 | 0.996 |
| | | | 50 | 0.941 | 0.950 | 0.595 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.998 | 0.758 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.3 | 0.50 | 50 | 25 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.000 | 0.001 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.5 | | 50 | 25 | 0.659 | 0.708 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.846 | 0.887 | 0.039 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.966 | 0.981 | 0.030 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 0.910 | 0.921 | 0.195 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 0.984 | 0.991 | 0.238 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 0.999 | 0.999 | 0.238 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.7 | | 50 | 25 | 0.999 | 0.999 | 0.940 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 0.992 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | 100 | 25 | 1.000 | 1.000 | 0.994 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 50 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

Table 9: Size of Panel Tests When the Model Allows for Low and Strong Cross Sectional Dependence. Intercept case.

A/Low

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.181 | 0.183 | 0.185 | 0.081 | 0.062 | 0.076 | 0.165 | 0.156 | 0.165 |
| | 50 | 0.199 | 0.190 | 0.194 | 0.083 | 0.068 | 0.076 | 0.210 | 0.204 | 0.200 |
| | 100 | 0.230 | 0.223 | 0.230 | 0.105 | 0.093 | 0.104 | 0.236 | 0.229 | 0.228 |
| 100 | 10 | 0.179 | 0.176 | 0.174 | 0.070 | 0.075 | 0.070 | 0.176 | 0.173 | 0.177 |
| | 25 | 0.179 | 0.173 | 0.181 | 0.083 | 0.091 | 0.085 | 0.185 | 0.180 | 0.188 |
| | 50 | 0.229 | 0.216 | 0.227 | 0.104 | 0.090 | 0.103 | 0.231 | 0.221 | 0.235 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.185 | 0.186 | 0.185 | 0.082 | 0.063 | 0.076 | 0.170 | 0.157 | 0.166 |
| | 50 | 0.200 | 0.191 | 0.197 | 0.085 | 0.069 | 0.077 | 0.210 | 0.207 | 0.202 |
| | 100 | 0.235 | 0.223 | 0.233 | 0.108 | 0.099 | 0.106 | 0.236 | 0.231 | 0.227 |
| 100 | 25 | 0.179 | 0.174 | 0.184 | 0.095 | 0.083 | 0.085 | 0.187 | 0.180 | 0.189 |
| | 50 | 0.230 | 0.217 | 0.228 | 0.104 | 0.090 | 0.106 | 0.231 | 0.223 | 0.238 |
| | 100 | 0.236 | 0.226 | 0.240 | 0.112 | 0.103 | 0.112 | 0.241 | 0.239 | 0.247 |

| T | N | $MH_{1m}(\mathcal{K})$ | $MH^d_{2m}(\mathcal{K})$ | $MH_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}_m$ | $M\mathcal{M}^{2*}_m$ | $M\mathcal{M}^{3*}_m$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.182 | 0.185 | 0.185 | 0.082 | 0.063 | 0.076 | 0.169 | 0.157 | 0.166 |
| | 50 | 0.200 | 0.191 | 0.196 | 0.085 | 0.069 | 0.077 | 0.210 | 0.207 | 0.202 |
| | 100 | 0.235 | 0.223 | 0.232 | 0.108 | 0.098 | 0.104 | 0.236 | 0.231 | 0.227 |
| 100 | 25 | 0.179 | 0.174 | 0.183 | 0.095 | 0.083 | 0.085 | 0.185 | 0.180 | 0.188 |
| | 50 | 0.229 | 0.217 | 0.227 | 0.104 | 0.090 | 0.105 | 0.231 | 0.222 | 0.236 |
| | 100 | 0.245 | 0.233 | 0.241 | 0.118 | 0.101 | 0.104 | 0.236 | 0.231 | 0.227 |

B/High

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.419 | 0.409 | 0.381 | 0.230 | 0.205 | 0.200 | 0.402 | 0.381 | 0.391 |
| | 50 | 0.477 | 0.458 | 0.433 | 0.255 | 0.246 | 0.215 | 0.425 | 0.406 | 0.425 |
| | 100 | 0.542 | 0.523 | 0.521 | 0.321 | 0.331 | 0.310 | 0.405 | 0.401 | 0.434 |
| 100 | 25 | 0.365 | 0.360 | 0.334 | 0.220 | 0.210 | 0.180 | 0.355 | 0.331 | 0.320 |
| | 50 | 0.431 | 0.423 | 0.401 | 0.255 | 0.246 | 0.215 | 0.429 | 0.416 | 0.409 |
| | 100 | 0.512 | 0.502 | 0.521 | 0.321 | 0.331 | 0.310 | 0.472 | 0.481 | 0.478 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.420 | 0.410 | 0.383 | 0.230 | 0.208 | 0.203 | 0.409 | 0.402 | 0.384 |
| | 50 | 0.478 | 0.458 | 0.434 | 0.257 | 0.246 | 0.215 | 0.443 | 0.406 | 0.427 |
| | 100 | 0.541 | 0.523 | 0.519 | 0.323 | 0.331 | 0.310 | 0.432 | 0.401 | 0.436 |
| 100 | 25 | 0.367 | 0.362 | 0.334 | 0.221 | 0.212 | 0.180 | 0.357 | 0.333 | 0.321 |
| | 50 | 0.428 | 0.414 | 0.380 | 0.208 | 0.251 | 0.232 | 0.399 | 0.391 | 0.372 |
| | 100 | 0.539 | 0.510 | 0.512 | 0.302 | 0.298 | 0.310 | 0.412 | 0.342 | 0.412 |

| T | N | $MH_{1m}(\mathcal{K})$ | $MH_{2m}(\mathcal{K})$ | $MH_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}_m$ | $M\mathcal{M}^{2*}_m$ | $M\mathcal{M}^{3*}_m$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.420 | 0.410 | 0.383 | 0.230 | 0.205 | 0.203 | 0.407 | 0.382 | 0.392 |
| | 50 | 0.477 | 0.459 | 0.434 | 0.256 | 0.215 | 0.246 | 0.442 | 0.427 | 0.406 |
| | 100 | 0.540 | 0.522 | 0.519 | 0.323 | 0.324 | 0.334 | 0.432 | 0.418 | 0.423 |
| 100 | 25 | 0.365 | 0.360 | 0.334 | 0.221 | 0.211 | 0.180 | 0.356 | 0.332 | 0.320 |
| | 50 | 0.428 | 0.414 | 0.379 | 0.251 | 0.230 | 0.207 | 0.399 | 0.391 | 0.371 |
| | 100 | 0.539 | 0.513 | 0.509 | 0.320 | 0.331 | 0.310 | 0.410 | 0.401 | 0.410 |

**Notes :** Empirical sizes corresponding to a 5% nominal size.

Table 10: Size of DeFactored Panel Tests. Intercept case

| T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^{1*}$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.066 | 0.067 | 0.066 | 0.065 | 0.063 | 0.063 | 0.059 | 0.056 | 0.061 |
| | 50 | 0.082 | 0.076 | 0.077 | 0.061 | 0.063 | 0.050 | 0.065 | 0.070 | 0.070 |
| | 100 | 0.080 | 0.077 | 0.080 | 0.059 | 0.058 | 0.059 | 0.071 | 0.068 | 0.074 |
| 100 | 25 | 0.064 | 0.062 | 0.063 | 0.070 | 0.072 | 0.068 | 0.066 | 0.065 | 0.065 |
| | 50 | 0.055 | 0.057 | 0.048 | 0.069 | 0.066 | 0.068 | 0.062 | 0.065 | 0.059 |
| | 100 | 0.057 | 0.055 | 0.501 | 0.068 | 0.067 | 0.057 | 0.063 | 0.062 | 0.053 |

| T | N | $MH^d_1(\mathcal{K})$ | $MH^d_2(\mathcal{K})$ | $MH^d_3(\mathcal{K})$ | $MH^d_1(\mathcal{K}^*)$ | $MH^d_2(\mathcal{K}^*)$ | $MH^d_3(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}$ | $M\mathcal{M}^{2*}$ | $M\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.069 | 0.067 | 0.070 | 0.067 | 0.066 | 0.064 | 0.062 | 0.058 | 0.064 |
| | 50 | 0.074 | 0.076 | 0.079 | 0.062 | 0.065 | 0.050 | 0.075 | 0.075 | 0.073 |
| | 100 | 0.080 | 0.077 | 0.081 | 0.060 | 0.058 | 0.059 | 0.073 | 0.069 | 0.074 |
| 100 | 25 | 0.064 | 0.062 | 0.063 | 0.069 | 0.072 | 0.068 | 0.065 | 0.065 | 0.064 |
| | 50 | 0.055 | 0.057 | 0.048 | 0.068 | 0.066 | 0.067 | 0.059 | 0.064 | 0.059 |
| | 100 | 0.057 | 0.054 | 0.051 | 0.067 | 0.065 | 0.057 | 0.061 | 0.061 | 0.053 |

| T | N | $MH_{1m}(\mathcal{K})$ | $MH_{2m}(\mathcal{K})$ | $MH_{3m}(\mathcal{K})$ | $MH_{1m}(\mathcal{K}^*)$ | $MH_{2m}(\mathcal{K}^*)$ | $MH_{3m}(\mathcal{K}^*)$ | $M\mathcal{M}^{1*}_m$ | $M\mathcal{M}^{2*}_m$ | $M\mathcal{M}^{3*}_{\mathrm{m}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 25 | 0.069 | 0.067 | 0.069 | 0.067 | 0.065 | 0.063 | 0.061 | 0.057 | 0.063 |
| | 50 | 0.072 | 0.076 | 0.079 | 0.061 | 0.064 | 0.050 | 0.075 | 0.075 | 0.073 |
| | 100 | 0.080 | 0.077 | 0.081 | 0.059 | 0.058 | 0.059 | 0.061 | 0.069 | 0.074 |
| | 25 | 0.064 | 0.064 | 0.064 | 0.070 | 0.072 | 0.069 | 0.067 | 0.066 | 0.066 |
| | 50 | 0.056 | 0.058 | 0.049 | 0.069 | 0.066 | 0.068 | 0.063 | 0.065 | 0.059 |
| | 100 | 0.057 | 0.055 | 0.051 | 0.068 | 0.067 | 0.057 | 0.063 | 0.062 | 0.053 |

**Notes :**Empirical sizes corresponding to a 5% nominal size.

Table 11: Power of Defactored Panel Tests. $I(0) \rightarrow I(1)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^1$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.400 | 0.408 | 0.405 | 0.161 | 0.171 | 0.088 | 0.350 | 0.364 | 0.335 |
| | | | 50 | 0.500 | 0.521 | 0.503 | 0.191 | 0.193 | 0.093 | 0.456 | 0.442 | 0.418 |
| | | | 100 | 0.684 | 0.672 | 0.657 | 0.255 | 0.264 | 0.103 | 0.624 | 0.625 | 0.595 |
| | | 100 | 25 | 0.834 | 0.821 | 0.798 | 0.454 | 0.485 | 0.201 | 0.821 | 0.811 | 0.737 |
| | | | 50 | 0.877 | 0.829 | 0.763 | 0.325 | 0.287 | 0.051 | 0.810 | 0.843 | 0.814 |
| | | | 100 | 0.912 | 0.893 | 0.865 | 0.574 | 0.291 | 0.034 | 0.900 | 0.893 | 0.861 |
| 0.5 | | 50 | 25 | 0.467 | 0.480 | 0.545 | 0.049 | 0.056 | 0.029 | 0.310 | 0.321 | 0.369 |
| | | | 50 | 0.688 | 0.661 | 0.740 | 0.053 | 0.057 | 0.041 | 0.134 | 0.140 | 0.154 |
| | | | 100 | 0.886 | 0.901 | 0.947 | 0.021 | 0.027 | 0.008 | 0.674 | 0.721 | 0.796 |
| | | 100 | 25 | 0.899 | 0.900 | 0.908 | 0.079 | 0.105 | 0.006 | 0.850 | 0.859 | 0.860 |
| | | | 50 | 0.555 | 0.506 | 0.494 | 0.082 | 0.078 | 0.013 | 0.437 | 0.404 | 0.319 |
| | | | 100 | 0.625 | 0.513 | 0.399 | 0.112 | 0.081 | 0.019 | 0.436 | 0.344 | 0.130 |
| 0.7 | | 50 | 25 | 0.297 | 0.315 | 0.419 | 0.027 | 0.026 | 0.019 | 0.153 | 0.166 | 0.249 |
| | | | 50 | 0.438 | 0.436 | 0.434 | 0.025 | 0.030 | 0.031 | 0.199 | 0.210 | 0.298 |
| | | | 100 | 0.593 | 0.633 | 0.817 | 0.007 | 0.008 | 0.004 | 0.275 | 0.317 | 0.484 |
| | | 100 | 25 | 0.849 | 0.864 | 0.943 | 0.006 | 0.008 | 0.002 | 0.635 | 0.688 | 0.822 |
| | | | 50 | 0.966 | 0.966 | 0.969 | 0.006 | 0.006 | 0.003 | 0.834 | 0.873 | 0.946 |
| | | | 100 | 0.998 | 0.978 | 0.999 | 0.053 | 0.052 | 0.072 | 0.919 | 0.956 | 0.997 |
| 0.3 | 0.50 | 50 | 25 | 0.678 | 0.625 | 0.583 | 0.064 | 0.068 | 0.039 | 0.394 | 0.372 | 0.283 |
| | | | 50 | 0.761 | 0.656 | 0.632 | 0.027 | 0.031 | 0.017 | 0.436 | 0.412 | 0.391 |
| | | | 100 | 0.876 | 0.777 | 0.748 | 0.007 | 0.007 | 0.004 | 0.611 | 0.575 | 0.560 |
| | | 100 | 25 | 0.997 | 0.992 | 0.984 | 0.076 | 0.082 | 0.042 | 0.940 | 0.925 | 0.854 |
| | | | 50 | 0.999 | 0.997 | 0.997 | 0.026 | 0.030 | 0.014 | 0.991 | 0.988 | 0.966 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.001 | 0.001 | 0.002 | 0.999 | 0.999 | 0.997 |
| 0.5 | | 50 | 25 | 0.609 | 0.594 | 0.455 | 0.025 | 0.021 | 0.014 | 0.698 | 0.679 | 0.644 |
| | | | 50 | 0.996 | 0.997 | 0.996 | 0.030 | 0.042 | 0.033 | 0.993 | 0.993 | 0.993 |
| | | | 100 | 0.999 | 0.999 | 0.999 | 0.018 | 0.016 | 0.027 | 0.998 | 0.999 | 0.999 |
| | | 100 | 25 | 0.947 | 0.920 | 0.949 | 0.009 | 0.014 | 0.000 | 0.692 | 0.665 | 0.608 |
| | | | 50 | 0.995 | 0.985 | 0.994 | 0.001 | 0.001 | 0.000 | 0.837 | 0.805 | 0.747 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.000 | 0.000 | 0.000 | 0.944 | 0.919 | 0.897 |
| 0.7 | | 50 | 25 | 0.174 | 0.144 | 0.112 | 0.059 | 0.047 | 0.055 | 0.312 | 0.295 | 0.263 |
| | | | 50 | 0.242 | 0.218 | 0.198 | 0.247 | 0.174 | 0.319 | 0.335 | 0.329 | 0.311 |
| | | | 100 | 0.347 | 0.284 | 0.260 | 0.429 | 0.372 | 0.481 | 0.503 | 0.559 | 0.570 |
| | | 100 | 25 | 0.528 | 0.502 | 0.464 | 0.010 | 0.009 | 0.009 | 0.439 | 0.426 | 0.374 |
| | | | 50 | 0.717 | 0.677 | 0.533 | 0.033 | 0.030 | 0.033 | 0.665 | 0.649 | 0.614 |
| | | | 100 | 0.947 | 0.928 | 0.882 | 0.063 | 0.053 | 0.099 | 0.853 | 0.834 | 0.806 |

Table 12: Power of Defactored Modified Panel Tests. $I(0) \to I(1)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_1^d(\mathcal{K})$ | $\text{MH}_2^d(\mathcal{K})$ | $\text{MH}_3^d(\mathcal{K})$ | $\text{MH}_1^d(\mathcal{K}^*)$ | $\text{MH}_2^d(\mathcal{K}^*)$ | $\text{MH}_3^d(\mathcal{K}^*)$ | $\text{M}\mathcal{M}^{1*}$ | $\text{M}\mathcal{M}^{2*}$ | $\text{M}\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.409 | 0.412 | 0.410 | 0.165 | 0.174 | 0.089 | 0.357 | 0.372 | 0.343 |
| | | | 50 | 0.551 | 0.568 | 0.559 | 0.189 | 0.200 | 0.092 | 0.473 | 0.498 | 0.465 |
| | | | 100 | 0.684 | 0.673 | 0.661 | 0.258 | 0.264 | 0.104 | 0.626 | 0.625 | 0.596 |
| | | 100 | 25 | 0.834 | 0.821 | 0.798 | 0.453 | 0.485 | 0.199 | 0.821 | 0.810 | 0.737 |
| | | | 50 | 0.877 | 0.829 | 0.814 | 0.324 | 0.287 | 0.305 | 0.882 | 0.856 | 0.834 |
| | | | 100 | 0.969 | 0.927 | 0.901 | 0.290 | 0.246 | 0.034 | 0.979 | 0.958 | 0.921 |
| 0.5 | | 50 | 25 | 0.477 | 0.486 | 0.546 | 0.051 | 0.057 | 0.029 | 0.317 | 0.327 | 0.377 |
| | | | 50 | 0.688 | 0.735 | 0.810 | 0.054 | 0.058 | 0.043 | 0.498 | 0.510 | 0.623 |
| | | | 100 | 0.888 | 0.902 | 0.947 | 0.021 | 0.027 | 0.008 | 0.682 | 0.712 | 0.796 |
| | | 100 | 25 | 0.898 | 0.900 | 0.907 | 0.079 | 0.105 | 0.005 | 0.848 | 0.858 | 0.859 |
| | | | 50 | 0.939 | 0.943 | 0.951 | 0.082 | 0.078 | 0.013 | 0.900 | 0.910 | 0.916 |
| | | | 100 | 0.993 | 0.999 | 0.999 | 0.112 | 0.081 | 0.019 | 0.967 | 0.987 | 0.978 |
| 0.7 | | 50 | 25 | 0.300 | 0.320 | 0.421 | 0.027 | 0.027 | 0.019 | 0.159 | 0.173 | 0.262 |
| | | | 50 | 0.453 | 0.487 | 0.578 | 0.025 | 0.030 | 0.032 | 0.212 | 0.245 | 0.324 |
| | | | 100 | 0.597 | 0.637 | 0.819 | 0.008 | 0.008 | 0.004 | 0.284 | 0.319 | 0.485 |
| | | 100 | 25 | 0.849 | 0.864 | 0.943 | 0.006 | 0.008 | 0.002 | 0.634 | 0.687 | 0.820 |
| | | | 50 | 0.966 | 0.966 | 0.969 | 0.006 | 0.006 | 0.003 | 0.834 | 0.873 | 0.946 |
| | | | 100 | 0.999 | 0.999 | 0.999 | 0.053 | 0.052 | 0.072 | 0.910 | 0.923 | 0.991 |
| 0.3 | 0.50 | 50 | 25 | 0.689 | 0.626 | 0.584 | 0.064 | 0.070 | 0.039 | 0.407 | 0.383 | 0.289 |
| | | | 50 | 0.762 | 0.659 | 0.635 | 0.028 | 0.032 | 0.018 | 0.589 | 0.579 | 0.465 |
| | | | 100 | 0.877 | 0.781 | 0.750 | 0.007 | 0.007 | 0.004 | 0.724 | 0.710 | 0.634 |
| | | 100 | 25 | 0.997 | 0.992 | 0.984 | 0.076 | 0.082 | 0.042 | 0.940 | 0.924 | 0.853 |
| | | | 50 | 0.999 | 0.997 | 0.997 | 0.026 | 0.030 | 0.014 | 0.991 | 0.987 | 0.963 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.001 | 0.002 | 0.001 | 0.999 | 0.997 | 0.997 |
| 0.5 | | 50 | 25 | 0.375 | 0.308 | 0.294 | 0.025 | 0.023 | 0.012 | 0.187 | 0.157 | 0.124 |
| | | | 50 | 0.510 | 0.489 | 0.467 | 0.027 | 0.021 | 0.015 | 0.298 | 0.246 | 0.210 |
| | | | 100 | 0.789 | 0.720 | 0.698 | 0.018 | 0.016 | 0.028 | 0.410 | 0.378 | 0.324 |
| | | 100 | 25 | 0.947 | 0.919 | 0.948 | 0.009 | 0.014 | 0.000 | 0.691 | 0.664 | 0.605 |
| | | | 50 | 0.995 | 0.985 | 0.994 | 0.001 | 0.001 | 0.000 | 0.837 | 0.802 | 0.746 |
| | | | 100 | 0.999 | 0.999 | 1.000 | 0.000 | 0.000 | 0.000 | 0.942 | 0.999 | 0.997 |
| 0.7 | | 50 | 25 | 0.176 | 0.145 | 0.114 | 0.063 | 0.051 | 0.057 | 0.116 | 0.095 | 0.067 |
| | | | 50 | 0.245 | 0.212 | 0.189 | 0.250 | 0.179 | 0.322 | 0.176 | 0.149 | 0.123 |
| | | | 100 | 0.349 | 0.286 | 0.246 | 0.430 | 0.375 | 0.481 | 0.207 | 0.196 | 0.178 |
| | | 100 | 25 | 0.528 | 0.501 | 0.464 | 0.010 | 0.009 | 0.009 | 0.439 | 0.426 | 0.374 |
| | | | 50 | 0.712 | 0.700 | 0.678 | 0.033 | 0.030 | 0.031 | 0.598 | 0.587 | 0.534 |
| | | | 100 | 0.887 | 0.854 | 0.819 | 0.063 | 0.053 | 0.098 | 0.810 | 0.778 | 0.730 |

Table 13: Power of Defactored Modified Min Panel Tests. $I(0) \to I(1)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_{1m}(\mathcal{K})$ | $\text{MH}_{2m}(\mathcal{K})$ | $\text{MH}_{3m}(\mathcal{K})$ | $\text{MH}_{1m}(\mathcal{K}^*)$ | $\text{MH}_{2m}(\mathcal{K}^*)$ | $\text{MH}_{3m}(\mathcal{K}^*)$ | $\text{M}\mathcal{M}_m^{1*}$ | $\text{M}\mathcal{M}_m^{2*}$ | $\text{M}\mathcal{M}_\text{m}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.403 | 0.410 | 0.407 | 0.162 | 0.173 | 0.089 | 0.354 | 0.368 | 0.340 |
| | | | 50 | 0.548 | 0.563 | 0.552 | 0.187 | 0.200 | 0.092 | 0.471 | 0.491 | 0.462 |
| | | | 100 | 0.684 | 0.672 | 0.661 | 0.258 | 0.264 | 0.103 | 0.626 | 0.625 | 0.595 |
| | | 100 | 25 | 0.836 | 0.821 | 0.799 | 0.458 | 0.488 | 0.201 | 0.824 | 0.811 | 0.737 |
| | | | 50 | 0.877 | 0.830 | 0.764 | 0.325 | 0.287 | 0.051 | 0.884 | 0.858 | 0.833 |
| | | | 100 | 0.968 | 0.929 | 0.902 | 0.295 | 0.246 | 0.034 | 0.971 | 0.967 | 0.921 |
| 0.5 | | 50 | 25 | 0.474 | 0.482 | 0.546 | 0.050 | 0.057 | 0.029 | 0.315 | 0.325 | 0.375 |
| | | | 50 | 0.683 | 0.731 | 0.798 | 0.053 | 0.057 | 0.042 | 0.496 | 0.508 | 0.621 |
| | | | 100 | 0.887 | 0.901 | 0.947 | 0.021 | 0.027 | 0.008 | 0.678 | 0.712 | 0.796 |
| | | 100 | 25 | 0.900 | 0.900 | 0.909 | 0.080 | 0.107 | 0.006 | 0.850 | 0.859 | 0.860 |
| | | | 50 | 0.941 | 0.943 | 0.953 | 0.082 | 0.079 | 0.013 | 0.900 | 0.910 | 0.916 |
| | | | 100 | 0.996 | 0.999 | 0.999 | 0.113 | 0.082 | 0.019 | 0.967 | 0.987 | 0.978 |
| 0.7 | | 50 | 25 | 0.297 | 0.317 | 0.421 | 0.027 | 0.027 | 0.019 | 0.156 | 0.168 | 0.259 |
| | | | 50 | 0.450 | 0.483 | 0.578 | 0.025 | 0.030 | 0.032 | 0.210 | 0.234 | 0.321 |
| | | | 100 | 0.597 | 0.636 | 0.819 | 0.007 | 0.008 | 0.004 | 0.282 | 0.318 | 0.484 |
| | | 100 | 25 | 0.849 | 0.865 | 0.943 | 0.006 | 0.008 | 0.002 | 0.640 | 0.693 | 0.823 |
| | | | 50 | 0.966 | 0.966 | 0.969 | 0.006 | 0.006 | 0.003 | 0.834 | 0.873 | 0.947 |
| | | | 100 | 0.998 | 0.999 | 0.999 | 0.054 | 0.052 | 0.073 | 0.912 | 0.922 | 0.990 |
| 0.3 | 0.50 | 50 | 25 | 0.682 | 0.625 | 0.583 | 0.064 | 0.069 | 0.039 | 0.404 | 0.378 | 0.287 |
| | | | 50 | 0.762 | 0.657 | 0.635 | 0.028 | 0.032 | 0.017 | 0.585 | 0.572 | 0.463 |
| | | | 100 | 0.876 | 0.779 | 0.748 | 0.007 | 0.007 | 0.004 | 0.785 | 0.708 | 0.635 |
| | | 100 | 25 | 0.997 | 0.993 | 0.984 | 0.076 | 0.082 | 0.042 | 0.941 | 0.928 | 0.859 |
| | | | 50 | 0.999 | 0.997 | 0.997 | 0.027 | 0.030 | 0.015 | 0.991 | 0.988 | 0.968 |
| | | | 100 | 1.000 | 1.000 | 1.000 | 0.001 | 0.002 | 0.001 | 0.999 | 0.999 | 0.997 |
| 0.5 | | 50 | 25 | 0.369 | 0.305 | 0.293 | 0.025 | 0.023 | 0.012 | 0.186 | 0.156 | 0.123 |
| | | | 50 | 0.500 | 0.482 | 0.466 | 0.026 | 0.021 | 0.014 | 0.297 | 0.246 | 0.209 |
| | | | 100 | 0.778 | 0.712 | 0.697 | 0.018 | 0.016 | 0.027 | 0.410 | 0.298 | 0.323 |
| | | 100 | 25 | 0.997 | 0.993 | 0.984 | 0.076 | 0.082 | 0.042 | 0.941 | 0.928 | 0.856 |
| | | | 50 | 0.995 | 0.985 | 0.994 | 0.001 | 0.001 | 0.000 | 0.841 | 0.807 | 0.750 |
| | | | 100 | 0.999 | 0.999 | 1.000 | 0.000 | 0.000 | 0.000 | 0.945 | 0.919 | 0.898 |
| 0.7 | | 50 | 25 | 0.175 | 0.144 | 0.114 | 0.062 | 0.050 | 0.057 | 0.116 | 0.095 | 0.067 |
| | | | 50 | 0.244 | 0.211 | 0.200 | 0.248 | 0.178 | 0.322 | 0.036 | 0.176 | 0.123 |
| | | | 100 | 0.347 | 0.285 | 0.265 | 0.429 | 0.374 | 0.481 | 0.207 | 0.210 | 0.179 |
| | | 100 | 25 | 0.529 | 0.503 | 0.464 | 0.010 | 0.010 | 0.009 | 0.441 | 0.426 | 0.375 |
| | | | 50 | 0.711 | 0.702 | 0.678 | 0.033 | 0.031 | 0.034 | 0.601 | 0.587 | 0.374 |
| | | | 100 | 0.875 | 0.857 | 0.818 | 0.064 | 0.053 | 0.101 | 0.810 | 0.778 | 0.730 |

Table 14: Power of Defactored Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $H_1(\mathcal{K})$ | $H_2(\mathcal{K})$ | $H_3(\mathcal{K})$ | $H_1(\mathcal{K}^*)$ | $H_2(\mathcal{K}^*)$ | $H_3(\mathcal{K}^*)$ | $\mathcal{M}^1$ | $\mathcal{M}^{2*}$ | $\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.039 | 0.044 | 0.030 | 0.252 | 0.260 | 0.366 | 0.154 | 0.165 | 0.202 |
| | | | 50 | 0.039 | 0.037 | 0.031 | 0.398 | 0.403 | 0.550 | 0.184 | 0.199 | 0.265 |
| | | | 100 | 0.009 | 0.014 | 0.009 | 0.577 | 0.596 | 0.757 | 0.244 | 0.284 | 0.457 |
| | | 100 | 25 | 0.003 | 0.009 | 0.001 | 0.851 | 0.875 | 0.954 | 0.616 | 0.681 | 0.817 |
| | | | 50 | 0.000 | 0.004 | 0.002 | 0.953 | 0.955 | 0.964 | 0.839 | 0.883 | 0.939 |
| | | | 100 | 0.070 | 0.055 | 0.079 | 0.993 | 0.995 | 0.999 | 0.934 | 0.971 | 0.989 |
| 0.5 | | 50 | 25 | 0.045 | 0.057 | 0.026 | 0.422 | 0.431 | 0.540 | 0.300 | 0.315 | 0.368 |
| | | | 50 | 0.062 | 0.071 | 0.049 | 0.632 | 0.657 | 0.687 | 0.523 | 0.543 | 0.589 |
| | | | 100 | 0.030 | 0.043 | 0.006 | 0.878 | 0.888 | 0.938 | 0.682 | 0.725 | 0.812 |
| | | 100 | 25 | 0.058 | 0.087 | 0.009 | 0.921 | 0.923 | 0.924 | 0.868 | 0.884 | 0.888 |
| | | | 50 | 0.067 | 0.062 | 0.010 | 0.965 | 0.967 | 0.970 | 0.921 | 0.943 | 0.951 |
| | | | 100 | 0.101 | 0.081 | 0.014 | 0.978 | 0.999 | 0.998 | 0.987 | 0.999 | 0.999 |
| 0.7 | | 50 | 25 | 0.138 | 0.155 | 0.095 | 0.386 | 0.382 | 0.384 | 0.346 | 0.365 | 0.357 |
| | | | 50 | 0.120 | 0.117 | 0.087 | 0.510 | 0.508 | 0.509 | 0.472 | 0.483 | 0.476 |
| | | | 100 | 0.264 | 0.304 | 0.114 | 0.667 | 0.647 | 0.634 | 0.638 | 0.633 | 0.600 |
| | | 100 | 25 | 0.444 | 0.452 | 0.203 | 0.815 | 0.808 | 0.768 | 0.793 | 0.773 | 0.704 |
| | | | 50 | 0.297 | 0.274 | 0.053 | 0.869 | 0.827 | 0.743 | 0.910 | 0.890 | 0.838 |
| | | | 100 | 0.280 | 0.222 | 0.022 | 0.930 | 0.910 | 0.887 | 0.987 | 0.975 | 0.929 |
| 0.3 | 0.50 | 50 | 25 | 0.057 | 0.052 | 0.073 | 0.151 | 0.126 | 0.105 | 0.089 | 0.083 | 0.050 |
| | | | 50 | 0.205 | 0.205 | 0.339 | 0.226 | 0.212 | 0.146 | 0.146 | 0.139 | 0.112 |
| | | | 100 | 0.409 | 0.341 | 0.448 | 0.375 | 0.301 | 0.170 | 0.254 | 0.180 | 0.171 |
| | | 100 | 25 | 0.010 | 0.012 | 0.010 | 0.523 | 0.500 | 0.467 | 0.427 | 0.415 | 0.366 |
| | | | 50 | 0.023 | 0.021 | 0.024 | 0.718 | 0.693 | 0.640 | 0.578 | 0.561 | 0.470 |
| | | | 100 | 0.055 | 0.036 | 0.087 | 0.836 | 0.825 | 0.798 | 0.741 | 0.721 | 0.653 |
| 0.5 | | 50 | 25 | 0.034 | 0.035 | 0.015 | 0.357 | 0.313 | 0.294 | 0.215 | 0.195 | 0.129 |
| | | | 50 | 0.021 | 0.021 | 0.015 | 0.756 | 0.678 | 0.633 | 0.374 | 0.328 | 0.223 |
| | | | 100 | 0.027 | 0.024 | 0.037 | 0.881 | 0.849 | 0.819 | 0.532 | 0.471 | 0.378 |
| | | 100 | 25 | 0.024 | 0.028 | 0.003 | 0.955 | 0.920 | 0.948 | 0.706 | 0.685 | 0.642 |
| | | | 50 | 0.002 | 0.003 | 0.000 | 0.995 | 0.991 | 0.997 | 0.836 | 0.805 | 0.745 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.953 | 0.936 | 0.915 |
| 0.7 | | 50 | 25 | 0.064 | 0.070 | 0.037 | 0.698 | 0.614 | 0.583 | 0.394 | 0.363 | 0.281 |
| | | | 50 | 0.022 | 0.018 | 0.007 | 0.747 | 0.774 | 0.719 | 0.525 | 0.491 | 0.361 |
| | | | 100 | 0.007 | 0.007 | 0.006 | 0.848 | 0.737 | 0.750 | 0.674 | 0.561 | 0.444 |
| | | 100 | 25 | 0.071 | 0.082 | 0.034 | 0.997 | 0.993 | 0.989 | 0.940 | 0.926 | 0.850 |
| | | | 50 | 0.117 | 0.017 | 0.021 | 0.998 | 0.998 | 0.997 | 0.992 | 0.988 | 0.969 |
| | | | 100 | 0.003 | 0.003 | 0.003 | 1.000 | 1.000 | 1.000 | 0.998 | 0.997 | 0.994 |

Table 15: Power of Defactored Modified Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_1^d(\mathcal{K})$ | $\text{MH}_2^d(\mathcal{K})$ | $\text{MH}_3^d(\mathcal{K})$ | $\text{MH}_1^d(\mathcal{K}^*)$ | $\text{MH}_2^d(\mathcal{K}^*)$ | $\text{MH}_3^d(\mathcal{K}^*)$ | $\text{M}\mathcal{M}^{1*}$ | $\text{M}\mathcal{M}^{2*}$ | $\text{M}\mathcal{M}^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.040 | 0.045 | 0.032 | 0.260 | 0.269 | 0.373 | 0.159 | 0.168 | 0.210 |
| | | | 50 | 0.039 | 0.039 | 0.032 | 0.401 | 0.414 | 0.567 | 0.192 | 0.134 | 0.179 |
| | | | 100 | 0.010 | 0.015 | 0.009 | 0.583 | 0.599 | 0.760 | 0.250 | 0.286 | 0.460 |
| | | 100 | 25 | 0.003 | 0.010 | 0.001 | 0.852 | 0.875 | 0.954 | 0.622 | 0.686 | 0.823 |
| | | | 50 | 0.000 | 0.004 | 0.002 | 0.953 | 0.955 | 0.964 | 0.839 | 0.884 | 0.939 |
| | | | 100 | 0.071 | 0.056 | 0.080 | 0.992 | 0.995 | 0.999 | 0.935 | 0.972 | 0.989 |
| 0.5 | | 50 | 25 | 0.048 | 0.058 | 0.026 | 0.428 | 0.438 | 0.547 | 0.311 | 0.324 | 0.375 |
| | | | 50 | 0.065 | 0.071 | 0.051 | 0.639 | 0.663 | 0.701 | 0.542 | 0.561 | 0.610 |
| | | | 100 | 0.030 | 0.043 | 0.006 | 0.880 | 0.889 | 0.939 | 0.688 | 0.729 | 0.815 |
| | | 100 | 25 | 0.059 | 0.088 | 0.009 | 0.923 | 0.923 | 0.915 | 0.885 | 0.886 | 0.890 |
| | | | 50 | 0.068 | 0.063 | 0.010 | 0.967 | 0.967 | 0.971 | 0.991 | 0.959 | 0.953 |
| | | | 100 | 0.102 | 0.082 | 0.014 | 0.610 | 0.518 | 0.389 | 0.443 | 0.999 | 0.999 |
| 0.7 | | 50 | 25 | 0.142 | 0.156 | 0.097 | 0.391 | 0.388 | 0.390 | 0.355 | 0.373 | 0.362 |
| | | | 50 | 0.120 | 0.118 | 0.087 | 0.515 | 0.510 | 0.511 | 0.478 | 0.490 | 0.478 |
| | | | 100 | 0.269 | 0.307 | 0.114 | 0.669 | 0.647 | 0.636 | 0.640 | 0.633 | 0.603 |
| | | 100 | 25 | 0.449 | 0.457 | 0.207 | 0.819 | 0.811 | 0.769 | 0.795 | 0.777 | 0.705 |
| | | | 50 | 0.298 | 0.277 | 0.054 | 0.869 | 0.828 | 0.799 | 0.797 | 0.790 | 0.780 |
| | | | 100 | 0.280 | 0.222 | 0.022 | 0.930 | 0.911 | 0.876 | 0.878 | 0.901 | 0.864 |
| 0.3 | 0.50 | 50 | 25 | 0.059 | 0.056 | 0.074 | 0.155 | 0.129 | 0.105 | 0.091 | 0.086 | 0.055 |
| | | | 50 | 0.264 | 0.208 | 0.339 | 0.229 | 0.214 | 0.146 | 0.151 | 0.142 | 0.103 |
| | | | 100 | 0.409 | 0.343 | 0.449 | 0.378 | 0.302 | 0.171 | 0.256 | 0.184 | 0.173 |
| | | 100 | 25 | 0.010 | 0.013 | 0.010 | 0.527 | 0.502 | 0.468 | 0.428 | 0.416 | 0.367 |
| | | | 50 | 0.023 | 0.022 | 0.024 | 0.722 | 0.695 | 0.641 | 0.579 | 0.562 | 0.470 |
| | | | 100 | 0.057 | 0.037 | 0.088 | 0.037 | 0.025 | 0.002 | 0.742 | 0.722 | 0.652 |
| 0.5 | | 50 | 25 | 0.039 | 0.038 | 0.016 | 0.359 | 0.317 | 0.296 | 0.224 | 0.199 | 0.132 |
| | | | 50 | 0.028 | 0.024 | 0.019 | 0.759 | 0.695 | 0.631 | 0.379 | 0.325 | 0.228 |
| | | | 100 | 0.029 | 0.024 | 0.038 | 0.891 | 0.854 | 0.819 | 0.532 | 0.473 | 0.378 |
| | | 100 | 25 | 0.024 | 0.028 | 0.003 | 0.957 | 0.922 | 0.951 | 0.716 | 0.687 | 0.646 |
| | | | 50 | 0.002 | 0.003 | 0.000 | 0.995 | 0.991 | 0.997 | 0.842 | 0.810 | 0.746 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 |
| 0.7 | | 50 | 25 | 0.064 | 0.072 | 0.037 | 0.702 | 0.625 | 0.590 | 0.407 | 0.373 | 0.287 |
| | | | 50 | 0.021 | 0.021 | 0.015 | 0.758 | 0.682 | 0.637 | 0.545 | 0.513 | 0.365 |
| | | | 100 | 0.007 | 0.008 | 0.006 | 0.850 | 0.737 | 0.723 | 0.678 | 0.643 | 0.445 |
| | | 100 | 25 | 0.072 | 0.082 | 0.035 | 0.997 | 0.993 | 0.989 | 0.944 | 0.927 | 0.855 |
| | | | 50 | 0.017 | 0.021 | 0.009 | 0.998 | 0.998 | 0.997 | 0.992 | 0.989 | 0.971 |
| | | | 100 | 0.003 | 0.003 | 0.003 | 1.000 | 1.000 | 1.000 | 0.998 | 0.997 | 0.995 |

Table 16: Power of Defactored Modified Min Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

| $\tau$ | $\sigma_\eta$ | T | N | $\text{MH}_{1m}(\mathcal{K})$ | $\text{MH}_{2m}(\mathcal{K})$ | $\text{MH}_{3m}(\mathcal{K})$ | $\text{MH}_{1m}(\mathcal{K}^*)$ | $\text{MH}_{2m}(\mathcal{K}^*)$ | $\text{MH}_{3m}(\mathcal{K}^*)$ | $\text{M}\mathcal{M}_m^{1*}$ | $\text{M}\mathcal{M}_m^{2*}$ | $\text{M}\mathcal{M}_m^{3*}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.3 | 0.10 | 50 | 25 | 0.040 | 0.045 | 0.032 | 0.256 | 0.263 | 0.371 | 0.157 | 0.167 | 0.209 |
| | | | 50 | 0.039 | 0.038 | 0.032 | 0.401 | 0.407 | 0.558 | 0.189 | 0.201 | 0.271 |
| | | | 100 | 0.010 | 0.015 | 0.009 | 0.581 | 0.597 | 0.760 | 0.249 | 0.284 | 0.460 |
| | | 100 | 25 | 0.003 | 0.009 | 0.001 | 0.852 | 0.875 | 0.954 | 0.617 | 0.682 | 0.819 |
| | | | 50 | 0.000 | 0.004 | 0.002 | 0.953 | 0.955 | 0.964 | 0.839 | 0.883 | 0.939 |
| | | | 100 | 0.070 | 0.055 | 0.080 | 0.994 | 0.996 | 0.999 | 0.934 | 0.971 | 0.989 |
| 0.5 | | 50 | 25 | 0.047 | 0.057 | 0.026 | 0.424 | 0.432 | 0.545 | 0.307 | 0.321 | 0.373 |
| | | | 50 | 0.063 | 0.071 | 0.051 | 0.634 | 0.659 | 0.681 | 0.529 | 0.551 | 0.591 |
| | | | 100 | 0.030 | 0.043 | 0.006 | 0.879 | 0.889 | 0.938 | 0.685 | 0.727 | 0.814 |
| | | 100 | 25 | 0.058 | 0.087 | 0.009 | 0.923 | 0.923 | 0.924 | 0.870 | 0.884 | 0.889 |
| | | | 50 | 0.068 | 0.063 | 0.010 | 0.972 | 0.972 | 0.970 | 0.920 | 0.942 | 0.949 |
| | | | 100 | 0.101 | 0.082 | 0.014 | 0.991 | 0.991 | 0.970 | 0.986 | 0.999 | 0.999 |
| 0.7 | | 50 | 25 | 0.140 | 0.156 | 0.097 | 0.389 | 0.386 | 0.388 | 0.354 | 0.369 | 0.361 |
| | | | 50 | 0.120 | 0.118 | 0.087 | 0.515 | 0.519 | 0.512 | 0.479 | 0.489 | 0.481 |
| | | | 100 | 0.269 | 0.305 | 0.114 | 0.669 | 0.647 | 0.635 | 0.640 | 0.633 | 0.602 |
| | | 100 | 25 | 0.444 | 0.452 | 0.203 | 0.815 | 0.811 | 0.769 | 0.795 | 0.776 | 0.704 |
| | | | 50 | 0.297 | 0.274 | 0.053 | 0.869 | 0.827 | 0.789 | 0.910 | 0.890 | 0.838 |
| | | | 100 | 0.280 | 0.222 | 0.022 | 0.930 | 0.910 | 0.876 | 0.987 | 0.975 | 0.929 |
| 0.3 | 0.50 | 50 | 25 | 0.059 | 0.054 | 0.074 | 0.154 | 0.129 | 0.105 | 0.090 | 0.086 | 0.055 |
| | | | 50 | 0.263 | 0.208 | 0.339 | 0.229 | 0.218 | 0.146 | 0.146 | 0.137 | 0.116 |
| | | | 100 | 0.409 | 0.342 | 0.449 | 0.378 | 0.302 | 0.171 | 0.255 | 0.184 | 0.172 |
| | | 100 | 25 | 0.010 | 0.012 | 0.010 | 0.523 | 0.501 | 0.467 | 0.428 | 0.415 | 0.366 |
| | | | 50 | 0.023 | 0.022 | 0.024 | 0.718 | 0.695 | 0.640 | 0.579 | 0.562 | 0.480 |
| | | | 100 | 0.055 | 0.036 | 0.088 | 0.836 | 0.821 | 0.799 | 0.744 | 0.722 | 0.663 |
| 0.5 | | 50 | 25 | 0.037 | 0.037 | 0.016 | 0.359 | 0.316 | 0.295 | 0.220 | 0.196 | 0.131 |
| | | | 50 | 0.028 | 0.024 | 0.019 | 0.756 | 0.679 | 0.631 | 0.381 | 0.329 | 0.225 |
| | | | 100 | 0.029 | 0.024 | 0.037 | 0.882 | 0.850 | 0.829 | 0.551 | 0.471 | 0.379 |
| | | 100 | 25 | 0.024 | 0.028 | 0.003 | 0.956 | 0.920 | 0.949 | 0.709 | 0.686 | 0.642 |
| | | | 50 | 0.002 | 0.003 | 0.000 | 0.995 | 0.991 | 0.997 | 0.839 | 0.806 | 0.745 |
| | | | 100 | 0.000 | 0.000 | 0.000 | 1.000 | 1.000 | 1.000 | 0.953 | 0.936 | 0.915 |
| 0.7 | | 50 | 25 | 0.064 | 0.071 | 0.037 | 0.699 | 0.621 | 0.584 | 0.402 | 0.368 | 0.283 |
| | | | 50 | 0.021 | 0.021 | 0.015 | 0.758 | 0.679 | 0.633 | 0.535 | 0.499 | 0.374 |
| | | | 100 | 0.007 | 0.008 | 0.006 | 0.850 | 0.737 | 0.722 | 0.689 | 0.573 | 0.459 |
| | | 100 | 25 | 0.072 | 0.082 | 0.035 | 0.997 | 0.993 | 0.989 | 0.940 | 0.926 | 0.851 |
| | | | 50 | 0.017 | 0.021 | 0.009 | 0.998 | 0.998 | 0.997 | 0.992 | 0.989 | 0.969 |
| | | | 100 | 0.003 | 0.003 | 0.003 | 1.000 | 1.000 | 1.000 | 0.998 | 0.997 | 0.995 |

Table 17: Inflation rate 1970.1-2007.3

| No cross-sectional dependence adjusted tests | | | | | |
|---|---|---|---|---|---|
| Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) |
| $H_1(\mathcal{K})$ | 1.664 (0.048) | $H_1(\mathcal{K}^*)$ | 408.373 (0.000) | $\mathcal{M}^{1*}$ | 352.873 (0.000) |
| $H_2(\mathcal{K})$ | −0.876 (0.809) | $H_2(\mathcal{K}^*)$ | 246.559 (0.000) | $\mathcal{M}^{2*}$ | 222.889 (0.000) |
| $H_3(\mathcal{K})$ | 1.415 (0.079) | $H_3(\mathcal{K}^*)$ | 549.627 (0.000) | $\mathcal{M}^{3*}$ | 436.048 (0.000) |
| $MH^d_1(\mathcal{K})$ | 1.731 (0.023) | $MH^d_1(\mathcal{K}^*)$ | 413.061 (0.000) | $M\mathcal{M}^{1*}$ | 357.636 (0.000) |
| $MH^d_2(\mathcal{K})$ | −0.844 (0.801) | $MH^d_2(\mathcal{K}^*)$ | 248.708 (0.000) | $M\mathcal{M}^{2*}$ | 225.168 (0.000) |
| $MH^d_3(\mathcal{K})$ | 1.474 (0.070) | $MH^d_3(\mathcal{K}^*)$ | 557.550 (0.000) | $M\mathcal{M}^{3*}$ | 443.112 (0.000) |
| $MH_{1_m}(\mathcal{K})$ | 1.667 (0.048) | $MH_{1_m}(\mathcal{K}^*)$ | 408.617 (0.000) | $M\mathcal{M}^{1*}_m$ | 353.187 (0.000) |
| $MH_{2_m}(\mathcal{K})$ | −0.874 (0.809) | $MH_{2_m}(\mathcal{K}^*)$ | 246.677 (0.000) | $M\mathcal{M}^{2*}_m$ | 223.052 (0.000) |
| $MH_{3_m}(\mathcal{K})$ | 1.418 (0.078) | $MH_{3_m}(\mathcal{K}^*)$ | 550.015 (0.000) | $M\mathcal{M}^{3*}_m$ | 436.187 (0.000) |
| Break Date : 1977.3 | | | | | |

| Cross-correlation dependence adjusted tests | | | | | |
|---|---|---|---|---|---|
| Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) | Test Statistics | test − value (p−values) |
| $H_1(\mathcal{K})$ | −0.846 (0.801) | $H_1(\mathcal{K}^*)$ | 55.396 (0.000) | $\mathcal{M}^{1*}$ | 46.288 (0.000) |
| $H_2(\mathcal{K})$ | −3.103 (0.999) | $H_2(\mathcal{K}^*)$ | 39.024 (0.000) | $\mathcal{M}^{2*}$ | 33.092 (0.000) |
| $H_3(\mathcal{K})$ | −0.946 (0.846) | $H_3(\mathcal{K}^*)$ | 70.435 (0.000) | $\mathcal{M}^{3*}$ | 54.639 (0.000) |
| $MH^d_1(\mathcal{K})$ | −0.829 (0.797) | $MH^d_1(\mathcal{K}^*)$ | 55.649 (0.000) | $M\mathcal{M}^{1*}$ | 46.547 (0.000) |
| $MH^d_2(\mathcal{K})$ | −3.097 (0.999) | $MH^d_2(\mathcal{K}^*)$ | 39.161 (0.000) | $M\mathcal{M}^{2*}$ | 33.236 (0.000) |
| $MH^d_3(\mathcal{K})$ | −0.933 (0.824) | $MH^d_3(\mathcal{K}^*)$ | 70.830 (0.000) | $M\mathcal{M}^{3*}$ | 54.994 (0.000) |
| $MH_{1_m}(\mathcal{K})$ | −0.844 (0.801) | $MH_{1_m}(\mathcal{K}^*)$ | 55.419 (0.000) | $M\mathcal{M}^{1*}_m$ | 46.321 (0.000) |
| $MH_{2_m}(\mathcal{K})$ | −3.102 (0.999) | $MH_{2_m}(\mathcal{K}^*)$ | 33.113 (0.000) | $M\mathcal{M}^{2*}_m$ | 33.113 (0.000) |
| $MH_{3_m}(\mathcal{K})$ | −0.945 (0.828) | $MH_{3_m}(\mathcal{K}^*)$ | 70.468 (0.000) | $M\mathcal{M}^{3*}_m$ | 54.682 (0.000) |
| Break Date : 1980.4 | | | | | |