Abstracts

Luigi Accardi (Roma)

MARKOV FIELDS

Fabio Bagarello (Palermo)

STOCK MARKETS AND QUANTUM DYNAMICS

We discuss a second quantization approach of a simplified stock market and we use the stochastic limit, as well as other perturbative approaches, to deduce the time behavior of the portfolio of the different traders of the market.

Alberto Barchielli (Milano)

QUANTUM TRAJECTORIES AND SQUEEZED LIGHT FROM A TWO-LEVEL ATOM

By the term squeezed light, usually one means some light with phase sensitive fluctuations, not understandable in terms of a classical electromagnetic theory with stochasticity. Measurement of squeezing is obtained by homodyne detection followed by a spectrum analyser. The mathematical (fully probabilistic) description of homodyne detection requires the theory of Quantum Continual Measurements or Quantum Trajectories and notions as positive operator valued measures, instruments..., typical of Quanum Probability. In this talk we illustrate the theory in a concrete case (a two-level atom stimulated by laser) and we show how to obtain the squeezing spectrum.

Viacheslaw P. Belavkin (Nottingham)

OPTIMAL DILATION AND CONTROL OF QUANTUM STOCHASTIC FLOWS

We consider a problem of optimal unravelling (dilation) of a given quantum state and a given dynamical semigroup under informational constraints for the purpose of optimal filtering and optimal feedback control. We show that the solution of such problems is reduced to a secondary-quantized Bellman-Jacoby type equation in the spaces of all unravelling with a given mutual information level. The solution of such equations can be found by a quantum dynamical programming method:

Alexander Belton (Cork)

DILATION OF QUANTUM DYNAMICAL SEMIGROUPS VIA RANDOM WALKS

Uniformly continuous quantum dynamical semigroups on von Neumann algebras or unital separable C* algebras are shown to have dilations which are vacuum cocycles, certain vacuum-adapted processes on Boson Fock space that are cocycles for the shift semigroup. These may be well approximated by quantum random walks; consequently, the dilations are necessarily $*$-homomorphic

B.V.Rajarama Bhat (Bangalore)

STABLE QUANTUM DYNAMICAL SEMIGROUPS

We look at quantum dynamical semigroups which decay to zero (in strong topology) as time tends to infinity. Such semigroups are called * stable*. We are interested in detecting stability by looking at the generator and also in understanding dilations of stable semigroups in comparison with the discrete situation. Some parts of the talk is based on joint work with Sachi Srivastava.

Raffaella Carbone (Pavia)

THE SPECTRAL GAP OF THE TWO-PHOTON ABSORPTION AND EMISSION SEMIGROUP

Joint work with F. Fagnola, J.C. Garcia and R. Quezada.

We compute the spectral gap for the two-photon absorption and emission semigroup, recently introduced by F. Fagnola and R. Quezada (Two photon absorption and emision process, *Infinite Dimens. Anal. Quantum Probab. Rel. Topics* 8, 2005, no.4).

Our interest is mainly concentrated on: (1) the invariant subspaces of the semigroup and (2) the relations of the quantum spectral gap with the spectral gap of some restriction of the semigroup to commutative algebras.

Vitonofrio Crismale (Bari)

ROTATION INVARIANT INTERACTING FOCK SPACES

It is well known that for boson, Fermi, boolean, free, monotone Fock spaces the distribution of the position operator is influenced only by the norm of the test function chosen, namely it is invariant as the test function runs over every sphere with given radius. This is the reason we call such spaces "rotation invariant".

On the other hand, this feature is not generally satisified by interacting Fock spaces: we will show exampels in which the law of the position operator depends either on the structure of the space or on the particular test function considered. As a consequence the rotation invariant interacting Fock spaces are a proper subset of interacting Fock spaces. In the talk we will present a characterization of this class.

Nicola Cufaro Petroni (Bari)

MIXTURES IN NON STABLE, SELF DECOMPOSABLE LÉVY PROCESSES

We analyze the Lévy processes produced by means of two interconnected classes of non stable, self decomposable distributions: the Variance Gamma and the Student laws. While the Variance Gamma family is closed under convolution, the Student one is not: this makes its time evolution more complicated. We prove that - at least for one particular type of Student processes suggested by recent empirical results, and for integral times - the distribution of the process is a mixture of other types of Student distributions, randomized by means of a new probability distribution. The mixture is such that along the time the asymptotic behavior of the probability density functions always coincide with that of the generating Student law. We put forward the conjecture that this can be a general feature of the Student processes. We finally analyze the Ornstein-Uhlenbeck process driven by our Lévy noises, and we find that the asymptotic behavior of the stationary distributions coincide with that of the background noise.

Franco Fagnola (Milano)

QUANTUM MARKOV SEMIGROUPS AND NON EQUILIBRIUM PHENOMENA

A classical Markov system with discrete state space E and evolution given by a semigroup $(T_t)_{t\geq 0}$ of positive, identity preserving, maps on $\ell^\infty(E)$ is in a stationary regime if there exists a probability distribution π on E which is T_t-invariant i.e. $\sum_{j\in E}\pi_j(T_t(f))_j = \sum_{j\in E}\pi_j f(j)$ for all $f\in\ell^\infty(E)$. If the system is irreducible, then π is strictly positive and unique.

The system is in *equilibrium* if π satisfies the detailed balance condition, namely the classical reversibility condition $\pi q_{ij} = \pi_j q_{ji}$, where $(q_{ij})_{i,j}$ denotes the transition matrix associated with the generator of $(T_t)_{t\geq 0}$. This condition is equivalent to self-adjointness of the operators T_t and of the generator in the Hilbert space $L^2(E,\pi)$ determined by the invariant measure.

In a non-equilibrium system the non-zero difference $\pi_i q_{ij} - \pi_j q_{ji}$ represents some current between the states i and j. The system continuously performs some cycles (transitions $i_1 \to i_2 \to \dots i_{n-1} \to i_n = i_1$) and the difference $\pi_i q_{ij} - \pi_j q_{ji}$ can be decomposed as

$$\pi_i q_{ij} - \pi_j q_{ji} = \sum_{c\in\mathcal{C}} w_c \left(J_c(i,j) - J_{c^-}(i,j)\right), \tag{1}$$

where the sum is over a family $\mathcal{C}$ of cycles c, c^- is the reverse c of the cycle c, J_c is the unitary passage matrix associated with the cycle c and $(w_c)_{c\in\mathcal{C}}$ is a family of positive constants (currents). Clearly $J_{c^-} = J_c^{-1}$ and the unitary matrices J_c define automorphisms of $\ell^\infty(E)$.

The evolution of a quantum open systems is usually described by a Quantum Markov Semigroup (QMS) on the algebra $\mathcal{B}(\mathsf{h})$ of bounded operators on a complex separable Hilbert space h. The generator of a norm continuous QMS can be written in the form

$$\mathcal{L}(x) = -\frac{1}{2}\sum_{\ell}\left(L_\ell^* L_\ell x - 2L_\ell^* x L_\ell + x L_\ell^* L_\ell\right) - i[H, x].$$

In this talk we consider a QMS with a faithful normal invariant state ρ

1. study the adjoint of a QMS in the Hilbert space associated with ρ,
2. characterise, in terms of the operators L_ℓ, H, the structure of generators of equilibrium QMS,
3. discuss the non-commutative generalisation of (1).

References

[1] L. Accardi, K. Imafuku: Dynamical detailed balance and local KMS condition for non-equilibrium states. `quant-ph/0209088`

[2] F. Fagnola, V. Umanità: Generators of detailed balance quantum Markov semigroups. Preprint, 2006.

[3] M. Qian, M. Qian: Circulation for Recurrent Markov Chains. *Probab. Theory Rel. Fields* (formerly *Z. Wahrscheinlichkeitstheorie verw. Gebiete*) **59** (1982), 203-210.

Francesco Fidaleo (Roma)

MARKOV STATES ON QUASI-LOCAL ALGEBRAS

We review the definition of Markov states on quasi–local algebras, their structure and the main properties on known models.

Paolo Gibilisco (Roma)

MEANS, MONOTONE FUNCTIONS, FISHER INFORMATION: FROM CLASSICAL DIVERSITY TO QUANTUM UNITY

The concepts of mean, monotone function and Fisher information have a very different origin and history in mathematics and statistics. In the XX century the quantum versions of the above objects were developed. Due to the results of Löwner, Kubo-Ando and Petz one gets that there exist natural bijections, given by the following formula

$$f \quad \longleftrightarrow \quad m_f(A,B) := A^{\frac{1}{2}} f(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}) A^{\frac{1}{2}} \quad \longleftrightarrow \quad \langle A, B\rangle_{\rho,f} := \mathrm{Tr}(A \cdot m_f(L_\rho, R_\rho)^{-1}(B)),$$

among operator monotone functions, operator means and quantum Fisher informations.

The various versions of quantum Fisher information are Riemannian metrics, on the manifold of mixed states, contracting under coarse graining. Petz and Sudar have shown that one can (radially) extend a quantum Fisher information to the pure states iff the associated function is *regular*, namely, iff $f(0) \neq 0$. All these extensions coincide, up to scalars, with the Fubini-Study metric.

I shall show that to every regular operator monotone function f one can associate a non-regular o. m. function $\tilde{f}$ by the formula

$$\tilde{f}(x) := \frac{1}{2}\left[(x+1) - (x-1)^2 \frac{f(0)}{f(x)}\right].$$

From this result one gets another formula useful to calculate the Riemannian metrics associated to f on commutators, namely

$$\frac{f(0)}{2}\langle i[\rho, A], i[\rho, B]\rangle_{\rho,f} = \mathrm{Cov}_\rho(A, B) - \mathrm{Tr}(m_{\tilde{f}}(L_\rho, R_\rho)(A_0)B_0).$$

From this last formula a new inequality follows that can be seen as a kind of uncertainty principle (see the talk by D. Imparato).

This is a joint work with D. Imparato and T. Isola.

References

[1] Gibilisco, P., D. Imparato and Isola, T., Uncertainty principle and quantum Fisher information II. To appear in *J. Math Phys.* 2007.

[2] Gibilisco, P. and Isola, T., Uncertainty principle and quantum Fisher information. *Ann. Inst. Stat. Math*, 59: 147–159, 2007.

Matteo Gregoratti (Milano)

CLASSICAL MARKOV CHAINS AND UNITARY QUANTUM STOCHASTIC FLOWS

We consider classical Markov chains, in continuous time, living in abelian algebras invariant for unitary quantum stochastic flows. We study a particular class of unitary flows and we find that the associated Markov chains are endowed with a very rich structure: the stochastic evolution of the classical Markov system is represented at the same time by a Markov chain and by a deterministic reversible time-homogeneous evolution of the system coupled with a classical environment with random initial state. Such a representation appears as the classical counterpart of the representation of a unitary quantum stochastic flow by a global Hamiltonian evolution.

Daniele Imparato (Torino)

UNCERTAINTY PRINCIPLE AND QUANTUM FISHER INFORMATION

The Heisenberg uncertainty principle for selfadjoint matrices can be stated as

$$\mathrm{Var}_\rho(A) \cdot \mathrm{Var}_\rho(B) \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2.$$

Schrödinger and Robertson improved that result adding the squared covariance

$$\mathrm{Var}_\rho(A) \cdot \mathrm{Var}_\rho(B) - \mathrm{Cov}_\rho(A,B)^2 \geq \frac{1}{4}|\mathrm{Tr}(\rho[A,B])|^2.$$

The lower bound of the above inequality is due to non-commutativity and appears whenever the commutator $[A,B]$ is non-trivial. It is natural to ask whether a similar phenomenon appears when the commutators $[\rho,A]$, $[\rho,B]$ are non-trivial. This is, indeed, the case.

Let $\mathcal{F}_{op}^r$ be the class of *regular* operator monotone functions (see the talk by P. Gibilisco) and denote by $\mathrm{Area}_\rho^f(u,v)$ the area spanned by two tangent vectors u,v with respect to the quantum Fisher information (which is a Riemanian metric) associated to f. In this talk we shall present the recently proved inequality

$$\mathrm{Var}_\rho(A) \cdot \mathrm{Var}_\rho(B) - \mathrm{Cov}_\rho(A,B)^2 \geq \frac{1}{4}\left(f(0) \cdot \mathrm{Area}_\rho^f(i[\rho,A], i[\rho,B])\right)^2 \qquad \forall f \in \mathcal{F}_{op}^r. \tag{2}$$

The standard uncertainty principle is a consequence of the Cauchy-Schwartz inequality. As a fundamental result in the Kubo-Ando theory of operator means, one proves that any operator mean is larger than the harmonic mean and smaller than the arithmetic mean

$$2(A^{-1}+B^{-1})^{-1} \leq m_f(A,B) \leq \frac{1}{2}(A+B).$$

Inequality (2) is just a consequence of this fundamental Kubo-Ando inequality.

This is a joint work with P. Gibilisco and T. Isola.

References

[1] Gibilisco, P., D. Imparato and Isola, T., Uncertainty principle and quantum Fisher information II. To appear in *J. Math Phys.* 2007.

[2] Gibilisco, P. and Isola, T., Uncertainty principle and quantum Fisher information. *Ann. Inst. Stat. Math*, 59: 147–159, 2007.

Alberto Lanconelli (Bari)

A NEW VERSION OF THE ITO'S FORMULA FOR THE STOCHASTIC HEAT EQUATION

We present an Ito's type formula for a linear one dimensional stochastic partial differential equation driven by a Gaussian white noise. The approach is based on the properties of the S-tranform and on the notion of Wick product. We also make a detailed comparison between our result and analogous formulas existing in the literature.

Yun Gang Lu (Bari)

ω-SINGLETON INDEPENDENCE

In this talk a new independence, namely -singleton independence, is investigated. We prove that any family of -singleton independent random variables must verify the singleton condition, non–crossing principle and factorization principle and they permit us to establish a central limit theorem. Moreover we introduce a certain interacting Fock space to represent the central limit said above.

Habib Rebbei (Tunis)

SHIFT OF QUANTUM WHITE NOISE

Michael Skeide (Campobasso)

CLASSICAL RESTRICTIONS: INVARIANCE OF COMMUATIVE SUBALGEBRAS UNDER QUANTUM MARKOV SEMIGROUPS

Classical Markov semigroup act on the algebra of random variables of a probability space. Often, a classical Markov semigroup can be identified as the restriction of a quantum Markov semigroup to a commutative subalgebra. In the past it such a possibility in many occasions gave rise to important information about the classical Markov semigroup.

In this talk we concentrate on the converse question: Given a quantum Markov semigroup on a von Neumann algebra, when does it allow for restrictions to a commutative subalgebra? In the case when this subalgebra is *maximal commutaive* we give a necessary and sufficient criterion in terms of the Hilbert GNS-module of the generator of the semigroup. We point out that our criterion generalizes Rebolledo's sufficient criterion for the case when the von Neumann algebra is $B(H)$.

This work is joint with Franco Fagnola.

Veronica Umanità (Milano)

THE CYCLE DECOMPOSITION OF A GENERIC QUANTUM MARKOV SEMIGROUP

Consider a classical Markov system with discrete state space E and evolution given by a semigroup $(T_t)_{t\geq 0}$ of positive, identity preserving, maps on $\ell^\infty(E)$. Let $(q_{ij})_{i,j\in E}$ be the transition rate matrix associated with the generator. Suppose that system is irreducible with a strictly positive and unique invariant measure π.

The system continuously performs some cycles (transitions $i_1 \to i_2 \to \dots i_{n-1} \to i_n = i_1$) and the difference $\pi_i q_{ij} - \pi_j q_{ji}$ can be decomposed as

$$\pi_i q_{ij} - \pi_j q_{ji} = \sum_{c\in\mathcal{C}} w_c \left(J_c(i,j) - J_{c^-}(i,j)\right), \tag{3}$$

where the sum is over a family $\mathcal{C}$ of cycles c, c^- is the reverse of the cycle c, J_c is the unitary passage matrix associated with the cycle c and $(w_c)_{c\in\mathcal{C}}$ is a family of positive constants (currents). Clearly $J_{c^-} = J_c^{-1}$ and the unitary matrices J_c define automorphisms of $\ell^\infty(E)$.

In this talk we illustrate the non-commutative generalisation of (3) for generic (uniformly continuous) QMSs on $\mathcal{B}(\mathsf{h})$ with h finite-dimensional, possessing an invariant faithful state ρ; these semigroups were introduced by Accardi and Imafuku .

Since the unitary passage matrix J_c defines a $*$-automorphism on the algebra $\ell^\infty(E)$, and every $*$-automorphism on $\mathcal{B}(\mathsf{h})$ is the form $x \mapsto U^*xU$ for some unitary operator U, a natural generalization of (1) consists in decomposing the deviation from the equilibrium $\Pi := \mathcal{L} - \widetilde{\mathcal{L}} - 2i[H, \cdot]$ as

$$\Pi(x) = \sum_{c\in\mathcal{C}} w_c \rho^{-1/2} \left(U_c^* x U_c - U_c x U_c^*\right) \rho^{-1/2},$$

where $\mathcal{C}$ and $(w_c)_{c\in\mathcal{C}}$ are families of cycles and positive constants respectively, and each U_c is an unitary operator associated to the cycle c. Actually, we will see that such a decomposition is not possible, but that we need exactly n unitary operators in order to characterize a cycle of length n; instead, if we denote by n_c the length of cycle c, we find the following representation

$$\Pi(x) = \sum_{c\in\mathcal{C}} w_c \rho^{-1/2} \left[\frac{1}{n_c} \sum_{j=1}^{n_c} \left((U_j^{(c)})^* x U_j^{(c)} - U_j^{(c)} x (U_j^{(c)})^*\right)\right] \rho^{-1/2}.$$

We would like to characterize the QMSs which admit such a decomposition: for example, a necessary condition to have (3) for the difference $\mathcal{L} - \widetilde{\mathcal{L}} - 2i[H, \cdot]$ is the commutation between the QMS and the modular automorphism, but it is not clear if this condition is also sufficient.

References.

[1] L. Accardi, F. Fagnola, S. Hachicha: Generic q-Markov Semigroups and Speed of Convergence of q-Algorithms. *Inf. Dim. Anal. Quantum Probab. Rel. Topics* **9** (2006) n.4, 567–594.

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[3] D.Q. Jiang, M. Qian, M.P. Qian: *Mathematical theory of nonequilibrium steady states: on the frontier of probability and dynamical systems.* Lecture notes in Mathematics 1833. Springer-Verlag, Berlin 2004.

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