

# Abstracts

Luigi Accardi (Roma)

## Renormalized nonlinear powers of white noise

Andrea Andrisani (Bari)

## Kinematics and Dynamics of processes solutions of Lévy SDE's

Works of Nelson, Follmer, Carlen and others shown the possibility of predicting events taking place in the domain of quantum phenomena by means of the mathematical tools of classical probability. There is a link between the Schrodinger equation and an appropriate S.D.E.  $dX(t) = b(X(t), t)dt + dW(t)$ , where the drift term  $b(x, t)$  is a Borel function which evolves in time depending on the initial condition, while the fluctuation term  $W(t)$  is a Wiener process. Infact we have  $\rho(x, t) = |\Psi(x, t)|^2$ , (Born postulate) where  $\rho$  is the density function of the solution of the S.D.E., and  $\Psi$  is the solution of the Schrodinger equation, for the same initial condition.

Now Wiener process can be considered as a particular case of a Lévy process, i.e. process with independent increments, stationary increments and stochastically continuous. Our work is to try to derive a generalization of the Schrodinger equation, i.e. a P.D.E. to which we can attribute a physical meaning of describing quantum phenomena since its solution were compatible with Born postulate, by substitution of a Wiener process by a more general Lévy process as the fluctuation term in the S.D.E., following Nelson approach.

Key words: Schrodinger equation, Levy processes, current velocity, osmotic velocity.

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Alberto Barchielli (Milano)

## The stochastic master equation: Some problems

One of the formulations of the theory of time-continuous measurements is based on two related stochastic differential equations, a linear and a nonlinear one. In quantum optics such a formulation is called “quantum trajectory theory” and the nonlinear SDE, which gives the evolution of the “a posteriori states” or “conditional states”, is called “stochastic master equation”. From the point of view of the theory of stochastic differential equations, the stochastic master equation presents some interesting, unusual problems: the law of the noise can depend on the solution of the equation itself, how to prove existence of solutions and uniqueness in law, how to prove that the solution is a random statistical operator when the initial condition is a state, how to prove consistency with the axiomatic formulation of quantum mechanics and to recover in some sense linearity, ... The aim of the talk is to formulate such problems and to present possible solutions.

Viacheslav P. Belavkin (Nottingham)

## Quantum Girsanov theorem revisited

We generalize the semi-quantum Girsanov theorem used to define quantum filtering dynamics with respect to the classical output noise to the noncommutative case of quantum temperature noise. The corresponding completely positive quantum stochastic dynamics is discussed in the predual space of a von Neumann algebra.

Alexander Belton (Lancaster)

## Quantum random walks with a non-faithful particle state

In the past few years much progress has been made in the analysis of quantum random walks. When the interacting particles lie in a vector state, the situation is now well understood. A recent example of Attal and Joye drew attention to the thermalisation phenomenon, which arises when the particle state is faithful. The general case for faithful states has recently been settled, and attention has turned to more general particle states. In this talk, we will see how a general normal state gives rise to behaviour which is intermediate between the two extreme (vector-state and faithful-state) situations.

Vitonofrio Crismale (Bari)

## $\alpha$ -Dependence and related central limit theorems

In the talk we introduce the notions of  $\alpha$ -dependent ( $\alpha \in \mathbb{N}$ ) classical, boolean, free and monotone algebraic random variables. They can be seen as a sort of generalization of the usual concepts of classical, boolean, free and monotone algebraic random variables, in the sense that, when  $\alpha = 0$ , one finds the relative notions coincide in all the cases but the monotone one. Moreover we present the  $\alpha$ -singleton condition and some further conditions on such  $\alpha$ -dependent random variables in order to obtain a central limit theorem in each case.

Nicola Cufaro Petroni (Bari)

## Lévy-Schrödinger equations

We analyze the extension of the well known relation between Brownian motion and Schrödinger equation to the family of the Lévy processes. We consider a Lévy-Schrödinger equation where the usual kinetic energy operator - the Laplacian - is generalized by means of a selfadjoint, pseudodifferential operator whose symbol is the logarithmic characteristic of an infinitely divisible, centered, symmetric law. The Lévy-Khintchin formula shows then how to write down this operator in an integro-differential form. When the underlying Lévy process is stable we recover as a particular case the fractional Schrödinger equation. A few examples are finally given and we find that there are physically relevant models - such as a form of the relativistic Schroedinger equation - that are in the domain of the non stable Lévy-Schrödinger equations.

Ameur Dhahri (Besancon)

## Repeated interactions, unitary random walks and the low density limit

We prove that the solution of the discrete evolution equation describing a repeated interaction model, which is directed by a discrete classical noise, is a random walk on a family of unitary operators. We further establish the connection between the normalization of order  $h$  in the repeated quantum interaction Hamiltonian and the low density limit.

Franco Fagnola (Milano)

## The quantum Markov semigroup approach to Quantum Fokker-Planck models

In this talk we discuss the quantum Markov semigroup approach to the study of Quantum Fokker-Planck models. In particular we study stationary states and the long time asymptotics for the quantum Fokker–Planck equation for an open system in a harmonic confinement potential, perturbed by a (large) sub-quadratic term. The methods developed by the speaker in collaboration with R.Rebolledo allowed us to solve several problems that remained open in the analysis of this model with a PDE approach based on the Wigner function.

Remus Floricel (Regina)

## A Wold decomposition for $E_0$ -semigroups

The purpose of this presentation is to discuss a purification procedure for  $E_0$ -semigroups that bears a conceptual resemblance to the classical Wold decomposition of an isometry.

Uwe Franz (Besancon)

## Bijection between idempotent states and coidealgebras of finite quantum groups

We present several characterisations of idempotent states on finite quantum groups. In particular, we give an isomorphism between the lattice of idempotent states and the lattice of coidealgebras, which describes the intermediate algebras in irreducible finite depth inclusions of  $\text{II}_1$  factors.

Matteo Gregoratti (Milano)

## Non commuting bosonic annihilators and quantum stochastic calculus

We rigorously introduce the singular annihilators  $a_i(+0)$  and  $a_j(-0)$  in the bosonic Fock space of quantum stochastic calculus. We define their natural domains and we show that, even if their products are densely defined, they typically do not commute. We discuss the relation between this annihilators, their non commuting property, and the quantum stochastic differential equation of Hudson-Parthasarathy.

Robin Hudsoni (Loughborough)

## Unitary bicocycles in non-Fock quantum stochastic calculus

Alberto Lanconelli (Bari)

## Hölder-Young type inequalities for the Gaussian Wick product

The purpose of this talk is to present several inequalities for the  $L^q$ -norm of the Wick product of two random variables. These estimates are based on a Jensen's type inequality for the Wick multiplication which we derive via a positivity argument. These results belong to a joint work with Aurel Stan

Yun Gang Lu (Bari)

## A new interacting Fock space

A new interacting Fock space (IFS in short) is studied in this paper. Combinatorially, the mixture moments of field operator of this IFS is driven by such a collections of pair partitions that is strictly smaller than the totality of all pair partitions and strictly bigger than the set of non-crossing pair partitions. From the point of view of analysis, this IFS is not standard in the sense that the interactions are not multiplier of a certain positive measurable function even if the IFS is based on a  $L^2$ -space.