Abstracts

Luigi Accardi (Roma)

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B.V.Rajarama Bhat (Bangalore)

Roots of states

Given a state ϕ on a unital C^*-algebra $\mathcal{A}$ we look at unital quantum dynamical semigroups $\{\tau_t\}_{t\geq 0}$ on $\mathcal{A}$ such that $\tau_{t_0}(\cdot) = \phi(\cdot)I$ for some $t_0 > 0$. We see that for the von Neumann algebra $\mathcal{B}(\mathcal{H})$, such quantum dynamical semigroups dilate to semigroups of unital $*$-endomorphisms (E_0-semigroups) in standard form and conversely all E_0-semigroups in standard form arise this way. I am talking about a specific problem in the context of a general theory.

Vitonofrio Crismale (Bari)

A De Finetti theorem on the CAR algebra

The symmetric states on a quasi local C*-algebra on the infinite set of indices J are those invariant under the action of the group of the permutations moving only a finite, but arbitrary, number of elements of J. The celebrated De Finetti Theorem describes the structure of the symmetric states (i.e. exchangeable probability measures) in classical probability. In the present talk we show an extension of De Finetti Theorem to the case of the CAR algebra, that is for physical systems describing Fermions. Namely, we show that the compact convex set of such states is a Choquet simplex, whose extremal (i.e. ergodic w.r.t. the action of the group of permutations previously described) are precisely the product states in the sense of Araki-Moriya. In order to do that, we present some ergodic properties naturally enjoyed by the symmetric states which have a self-containing interest. The talk is based on a joint work with Francesco Fidaleo (Department of Mathematics, University of Tor Vergata, Roma).

Franco Fagnola (Milano)

Support projections of states of quantum open systems and non-commutative Lévy-Austin-Ornstein theorems

Support projection of classical and quantum probability densities of Markov processes play an important role in the study of stochastic evolutions determining those states that can be visited with strictly positive probability (we refer to [3] for quantum Markov processes).

The classical Lévy-Austin-Ornstein theorem [2] proves that the support projection of the probability density of a time continuous Markov chain is constant for strictly positive times. Quantum versions of this result are much more difficult because the typical quantum Markov process involves jumps and diffusion at the same time. However, they are unavoidable when studying the entropy production of quantum Markov semigroups describing non-equilibrium systems ([1]).

In this talk, based on a work in progress with R. Rebolledo, we consider a state ρ_0 of an open quantum system undergoing a Markovian time homogeneous evolution, characterise support projections of states ρ_t at time t and show an extension of the classical Lévy-Austin-Ornstein theorem. As a byproduct we find a Hörmander type condition for irreducibility of quantum Markov semigroups.

We also discuss some open problems and conjectures.

References

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[2] P. Lévy, Systèmes markoviens et stationnaires. Cas dénombrable. Ann. Sci. Ecole Norm. Sup. **68** (1951), 327–381. `www.numdam.org/item?id= ASENS_1951_3_68_327_0`

[3] V. Umanità, Classification and Decomposition of Quantum Markov Semigroups. *Probab. Theory Relat. Fields* **134**, (2006) 603–623, DOI 10.1007/s00440-005-0450-7.

Francesco Fidaleo (Roma)

The Bose-Einstein condensation on inhomogeneous networks

We present the new and unexpected results concerning the Bose–Einstein Condensation for the *Pure Hopping Model*, describing arrays of Josephson junctions located on non homogeneous networks. The amenable and the non amenable cases will be considered. The amenable case is summarized in the following table.

	ρ_c	R/T	d_G	d_{PF}	BEC	ρ–BEC
Z^d, $d < 3$	∞	R	d	d	no	no
Z^d, $d \geq 3$	$< \infty$	T	d	d	yes	yes
stargraph	$< \infty$	R	1	0	no	no
$\mathrm{Z}^d \dashv \mathbb{Z}$, $d < 3$	$< \infty$	R	$d+1$	d	no	no
$\mathrm{Z}^d \dashv \mathbb{Z}$, $d \geq 3$	$< \infty$	T	$d+1$	d	yes	no
N	∞	T	1	3	yes	no
$\mathrm{N} \dashv \mathbb{Z}^2$	$< \infty$	T	3	3	yes	yes

Here, $A \dashv B$ is the comb–shaped graph whose base–point is A, ρ_c is the critical density, R/T denotes the transience/recurrence of the adjacency, BEC (ρ–BEC) denotes the existence of locally normal states exhibiting BEC (exhibiting BEC at any mean density $\rho > \rho_c$).

The graphs under investigation are obtained by adding density zero perturbations to periodic amenable networks, and homogeneous Cayley Trees. The resulting (purely topological) model is described by a one particle Hamiltonian which is, up to an additive constant, the opposite of the adjacency operator on the graph. In the condensation regime, the particles condensate on the perturbed graph, even in the configuration space due to non homogeneity. Roughly speaking, the system undergoes a sort of "dimension transition". We show for both amenable and non amenable situations, that it is enough to perturb in a negligible way the original graph in order to obtain a new network whose mathematical and physical properties dramatically change. The appearance of the *Hidden Spectrum* near the zero of the Hamiltonian, or equivalently below the norm of the adjacency. The latter is related to the value of the critical density and then with the appearance of the condensation phenomena. The investigation of the *recurrence/transience character* of the adjacency, which is connected to the possibility to construct locally normal states exhibiting the Bose–Einstein condensation. Finally, the study of the *volume growth of the wave function* of the ground state of the Hamiltonian, which is nothing but the generalized Perron–Frobenius eigenvector of the adjacency. This Perron–Frobenius weight describes the spatial distribution of the condensate and its shape is connected with the possibility to construct locally normal states exhibiting the Bose–Einstein condensation at a fixed density greater than the critical one.

Uwe Franz (Besancon)

On the quantum symmetry group of a complex Hadamard matrix

To each complex Hadamard matrix one can associate a unique “quantum symmetry group” (or quantum permutation group, i.e. a subgroup of the free permutation compact quantum group S_N^+). Many questions about the subfactors and planar algebras associated to a Hadamard matrix have an equivalent formulations in terms of its quantum symmetry group. In my talk I will present a probabilistic approach to characterising this quantum symmetry group and study several examples in small dimension. Based on joint work with Teodor Banica, Franz Lehner, and Adam Skalski.

Malte Gerhold (Greifswald)

Finite dimensional subproduct systems

We will reduce the question, which sequences of dimensions are possible for a finite dimensional subproduct systems of Hilbert spaces, to a purely combinatorial question related to the number of words not containing certain subwords. To this end, we introduce the notion of “word systems”, which are a kind of analogue to subproduct systems, where Hilbert spaces are replaced by sets and tensor products are replaced by cartesian products. We will prove that for a given sequence of natural numbers, there exists a subproduct system with this sequence of dimensions iff there exists a word system with this sequence of cardinalities.

Robin Hillier (Roma)

The super-Virasoro algebra in conformal quantum field theory

We briefly introduce the Virasoro Lie algebra and some of its supersymmetric extensions. In quantum probability they appear in the description of white noise, but here we will rather focus on applications in conformal quantum filed theory. We recall the definition of conformal nets, deal with the construction of examples based on the Virasoro algebra, classifications, representations, and (noncommutative) geometric interpretations.

Based on joint work with Carpi, Kawahigashi, Longo, Xu: arXiv: math.OA/1207.2398

Robin Hudson (Loughborough)

Moments of classical and quantum Lévy area

I will describe methods of calculating the moments of Lévy’s stochastic area in classical and quantum (Fock and non-Fock) quantum stochastic calculus.

Martin Lindsay (Lancaster)

Stochastic dilation of minimal quantum dynamical semigroups

A minimal quantum dynamical semigroup on the full algebra of all operators on a Hilbert space $\mathfrak{h}$, in the sense of Davies (after Kato and Feller), is specified by its infinitesimal data — a pair of operators consisting of the generator K of a contractive C_0-semigroup on $\mathfrak{h}$ and an operator L from $\mathfrak{h}$ into $\mathfrak{h} \otimes \mathsf{k}$, for another Hilbert space k, which together satisfy a dissipativity relation.

In this talk I show how, under a sectorial condition on K, the quantum dynamical semigroup may be 'dilated' by means of a quantum stochastic contraction cocycle on $\mathfrak{h}$ driven by quantum noise with multiplicity space k. Necessary and sufficient conditions will be given for the dilation to be an E-semigroup (respectively, an E_0-semigroup), in terms of (two families of) $(d+1)$ associated semigroups where $d = \dim \mathsf{k}$.

What is involved here is an infinitesimal analysis of quantum stochastic contraction cocycles extending that of Lindsay, Parthasarathy and Wills for cocycles whose expectation semigroup is norm-continuous (which corresponds to K and L being bounded), to a complete characterisation of the class of quantum stochastic contraction cocycles whose expectation semigroup is holomorphic. The cocycles need no longer satisfy a quantum stochastic differential equation; their infinitesimal description is instead by means of quadratic forms and this is obtained by Kato-style perturbation arguments.

This is joint work with Kalyan B. Sinha. It is supported by the UKIERI Research Collaboration Network *Quantum Probability - Noncommutative Geometry - Quantum Information*

Yun Gang Lu (Bari)

A class of Gaussian Fock Spaces

We give a classification of a class of Gaussian Fock Spaces and calculate the distribution of the field operator in some particular case.

Lukas Neumann (Innsbruck)

Wigner Fokker Planck equation: critical and more critical cases

The Wigner Fokker Planck equation is a model for the interaction of particles with a heat bath. We study the irreducibility of the evolution in order to establish the existence of a unique stationary state. It turns out that the behaviour depends on the precise parameter regime – more precisely the relation between friction and diffusion. Indeed there are special cases when the evolution is not irreducible however in the generic case irreducibility holds. An overview of the results in different regimes will be given and a special case that is not resolved yet will be discussed.

These are results of joint work with A. Arnold (Vienna) and F. Fagnola (Milano).

Kimiaki Saito (Nagoya)

Infinite dimensional analysis based on higher order derivatives of white noise

In this talk we present an infinite dimensional analysis based on higher order derivatives of white noise. This analysis is associated with the exotic Laplacians and the Gross-Volterra Laplacians with orthonormal bases consisting from generalized functions. The exotic Laplacian acting on white noise distributions connects with the Lévy Laplacian acting on functionals of higher order derivative of white noise by the second quantization of the adjoint operator of the higher order differentiation. This result also gives a generalization of the Accardi-Smolyanov theorem in the higher order Cesàro mean. The relationship implies an interesting result on an infinite dimensional stochastic process generated by the exotic Laplacian. We also discuss the stochastic analysis related to the infinite dimensional stochastic process and give a relationship with the generalized harmonic analysis.

References

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[4] U. C. Ji and K. Saitô: *A similarity between the Gross Laplacian and the Lévy Laplacian*, Infin. Dimen. Anal. Quantum Probab. Rel. Top. **10** (2007), 261–276.

Emanuela Sasso (Genova)

Environment induced decoherence for Markovian evolutions

Decoherence is a phenomenon due to the interaction of the quan- tum system with its environment, and it has been accepted as the mechanism responsible for the appearance of classicality in quantum dynamics. One of the most popular mathematical definitions of decoherence was introduced by Blanchard and Olkiewicz: the key point is that, if decoherence takes place, the algebra describing the system can be decomposed in a maximal subalgebra M1 on which the evolution is reversible (the decoherence-free sector), and in a complementary subspace M2 on which the dynamics vanishes in time. According to this denition, we are trying to determine necessary and sufficient conditions for decoherence of quantum Markov evolutions In the case of semigroups acting on matrix algebra, this condition is related to the spectral analysis of the generator L of the semigroup and is easily stated. At the contrary, when the dimension is infinite, the characterization looks more complicated.

Michael Schürmann (Greifswald)

Schönberg correspondence on dual groups

As in the classical case of Lévy processes on a group, Lévy processes on a Voiculescu dual group are constructed from conditionally positive functionals. It is essential for this construction that Schoenberg correspondence holds for dual groups: The exponential of a conditionally positive functional is a convolution semigroup of states.

Roland Speicher (Saarbrücken)

Sharp bounds for sums associated to graphs of matrices

We provide a simple algorithm for finding the optimal upper bound for sums of products of matrix entries of the form $S_\pi(N) := \sum_{j_1,\ldots,j_{2m}=1}^{N} t^1_{j_1 j_2} t^2_{j_3 j_4} \ldots t^m_{j_{2m-1} j_{2m}}$ where some of the summation indices are constrained to be equal. The upper bound is easily obtained from a graph G associated to the constraints in the sum.

Michael Skeide (Campobasso)

Von Neumann modules — and related topics

Von Neumann modules have been "invented" in 2000 (preprint 1997). Every (pre-)Hilbert Hilbert module over a concrete (pre-)C^*-algebra $B \subset B(G)$ embeds in an essentially unique way into $B(G,H) \subset B(G \oplus H)$. There are many ways to make sure that a subspace E of $B(G,H)$ is a concrete Hilbert module over a $*$-subalgebra of $B(G)$. No matter how this is attained, E is a von Neumann module if it is strongly closed in $B(G,H)$. This is opposed with the definition of W^*-modules as self-dual Hilbert modules over a W^*-algebra—and, in fact, equivalent for every way we turn the W^*-algebra into a von Neumann algebra by choosing a faithful normal (nondegenerate) representation. We give an account what happened to and with von Neumann modules.

Wilhelm von Waldenfels (Heidelberg)

My way with quantum probability