Abstracts for the Workshop

Dilations, Endomorphism Semigroups, and their Classification by Product Systems

Greifswald, June 18 - 21, 2003

B.V.R. Bhat: Survey "Product systems and endomorphism semigroups from dilation" An E-semigroup is a one parameter semigroup of $*$-endomorphisms of a C^*-algebra. We are especially interested with E-semigroups of the algebra of all bounded operators on a Hilbert space. In late eighties R.T. Powers introduced an index theory for E-semigroups in an effort to classify them. W. Arveson showed as to how to attach a product system of Hilbert spaces with an E-semigroup. This led to the 'type classification' of E-semigroups. Here we give an introduction to this subject. We also explain the connections with dilation theory of quantum dynamical semigroups.

B.V.R. Bhat: Different pictures of units The concept of units or intertwining semigroups is very basic for classification of E-semigroups. Here we give several different descriptions of units and see as to what happens to units under dilation.

Rolf Gohm: Shift cocycles and scattering Consider the endomorphism R of $\mathcal{B}(\mathcal{H})$ which corresponds to a shift in an infinite tensor product of Hilbert spaces. We study the question when certain cocycle perturbations of amplifications of R are conjugate to R and show how this is related to a scattering theory introduced by Kümmerer and Maassen.

Volkmar Liebscher: Random sets and invariants for E_0 semigroups

Martin Lindsay: Quantum stochastic dilation

Michael Skeide: Survey "Lecture on Hilbert modules" We give a brief introduction to Hilbert modules. We put emphasis on the representation theory of $B^a(E)$, the C^*–algebra of all adjointable operators on a Hilbert module. After repeating the construction for the case when E has a unit vector, we concentrate on von Neumann modules. We illustrate the new approach to representation theory via the commutant of a von Neumann bimodule (also called a W^*–correspondence). Like in the case of Hilbert spaces the representation theory of $B^a(E)$ is the basic tool in the construction of product systems from E_0–semigroups on $B^a(E)$.

Michael Skeide: Constructions of product systems (of Hilbert modules) We illustrate several constructions of product systems and explain their interrelation. These are: Product systems vom classical Markov processes. Product systems from quantum Markov semigroups. Product systems from quantum Lévy processes. Product systems from E_0–semigroups on $B^a(E)$ in the case of unital Hilbert modules and in the case of (full) von Neumann modules. Representations of product systems by operators on a Hilbert space and the analogue of Arveson's spectral algebra.

Baruch Solel: Duality of C^*-correspondences and its applications A C^*-correspondence is, roughly, a bimodule over a C^*-algebra A which is also a right Hilbert C^*-module (i.e. it is equipped with an A-valued inner product). Such correspondences proved to be useful in the study of completely positive maps on A (and semigroups of such maps). In the talk, the algebra A will, in fact, be a von Neumann algebra. I will present the concept of duality for such correspondences and discuss some of its properties.

Every correspondence E gives rise to a (non selfadjoint) operator algebra denoted $H^\infty(E)$. When $A = E = \mathbb{C}$, we get the classical Hardy algebra on the unit disc.

We will show that the algebras associated with E and with its dual are the commutant of each other (in some canonical representation). We shall see how to view elements of the algebra associated with E as functions on the open unit ball of the dual. Exploiting this point of view, I shall present an interpolation result and, time permiting, results concerning contractive representations of the algebra. All these results can be viewed as generalizing known results for the Hardy algebra on the disc.

The talk is based on results obtained jointly with Paul S. Muhly.

Raman Srinivasan: TBA

Boris Tsirelson: 1. Talk "From off-white noises to type III systems" For the white noise, the spectral density is constant, and the future is independent of the past. If the spectral density is not too far from being constant, the dependence between the past and the future can be eliminated by an equivalent measure change dependent on the cut point (the present). Such a stationary Gaussian random process is called an off-white noise.

The white noise leads to a type I product system via the Fock space. Every off-white noise whose spectral density is bounded and tends to 0 at infinity, leads to a type III product system via a similar exponential construction that converts a continuous sum into a continuous product. However, the continuous sum is non-orthogonal.

Any two slowly varying spectral densities with unbounded ratio lead to two non-isomorphic product systems. A question of W. Arveson is thus answered: we get a continuum of mutually non-isomorphic product systems of type III.

Boris Tsirelson: 2. Talk "From off-Poisson noises to type II-0 systems" The Poisson point process is a stationary random subset of the real line such that the future is independent of the past, and the subset is locally finite (almost surely). It leads to a type I product system via the Fock space.

By an off-Poisson noise I mean a stationary random nowhere dense closed subset of the real line such that the dependence between the past and the future can be eliminated by an equivalent measure change dependent on the cut point (the present). Such a set need not be locally finite; moreover, it can be perfect (with no isolated points) almost surely. Then it leads to a product system of type II-0.

The Hausdorff dimension of the random set is usually non-random, it is a numerical characteristic of the off-Poisson noise. Any two such noises having different Hausdorff dimensions lead to two non-isomorphic product systems. A question of W. Arveson is thus answered: we get a continuum of mutually non-isomorphic product systems of type II-0.

A finer control of the random set gives product systems asymmetric in time.

Joachim Zacharias: Algebras associated to product systems There are several types of Banach and other algebras associated to product systems of Hilbert spaces and Hilbert modules e.g. non-selfadjoint algebras, Banach $*$-algebras, C^*-algebras and dense subalgebras thereof which may be viewed as domains of unbounded derivations. For instance the non-selfadjoint ones are often nonisomorphic whereas the C^*-algebras associated to different product systems tend to be all the same. We review some of the techniques available to study such algebras (mainly the C^*-algebras) including the computation of their K-theory and techniques used to establish simplicity and we try to point out interesting open questions.